

Class worksheet 8: Mathematical analysis 1

April 24, 2024

Name: _____

This is just to practice, no points are awarded. $\mathbb{N} = \{1, 2, \dots\}$, log with unspecified base is the natural logarithm.

1. Apply the inverse function rule to compute the derivatives of

- (a) \sqrt{x}
- (b) $\log x$
- (c) $\arcsin x$
- (d) $\arctan x$

2. Prove the following inequalities.

- a) For all $a, b \in \mathbb{R}$ we have $|\cos a - \cos b| \leq |a - b|$.
- b) For all $x \in (-1, \infty)$ we have $x + 1 \geq \exp(\frac{x}{1+x})$.

3. (*) Prove that if the functions f and g are continuous on $[a, b]$ and differentiable on (a, b) , then for some $c \in (a, b)$ we have

$$(f(b) - f(a))g'(c) = (g(b) - g(a))f'(c).$$

Hint: consider the function $h(x) = f(x) - rg(x)$, where r is chosen in order to achieve $h(a) = h(b)$.

The above fact is known as Cauchy's mean value theorem. It implies L'Hospital's theorem. Can you see how?