Class worksheet 7: Mathematical analysis 1

April 17, 2024

Name: _____

This is just to practice, no points are awarded. $\mathbb{N} = \{1, 2, \dots\}$, log with unspecified base is the natural logarithm.

1. Differentiate

   (a) $\frac{x-1}{x+1}$

   (b) $(\frac{x-1}{x+1})^2$

   (c) $\sqrt{\frac{x+2}{x+1}}$

   (d) $\sqrt{x+\sqrt{x+\sqrt{x}}}$

   (e) $\sqrt{1-\ln x}$

   (f) $x^2\sqrt{\ln x - 1}$

   (g) $\ln(\ln(\ln x))$

   (h) $\ln(1+e^{-x^2})$

   (i) $\sin\sqrt{x+1}$

   (j) $(\cos(2x))^5$

   (k) $\ln(\arctan(\sqrt{x}))$

   (l) x^x

   (m) $x^{\ln x}$

   (n) $x \cdot |x|$

   (o) $|(\sin x)^3|$

   (p) $|(x-1)^2(x+1)^3|$

   (q) The function f given by $f(x) = \begin{cases} x^2\sin(1/x), & \text{for } x \neq 0, \\ f(0) = 0. \end{cases}$

Please turn over.

2. Determine *from the definition* the derivatives of the following functions

   (a) x^n, $n \in \mathbb{N}$

   (b) $\sqrt{x}$

   (c) $\sin x$

   (d) $\log x$

   (e) e^x

3. Determine the local and global extrema (i.e., maxima and minima) of the following functions.

   a) $f : \mathbb{R} \setminus \{1\} \to \mathbb{R}$,
   $$f(x) = \frac{|2x-1|}{(x-1)^2}.$$

   b) $g : \mathbb{R} \setminus \{-1, 1\} \to \mathbb{R}$,
   $$g(x) = \exp\left(\frac{x^2+1}{x^2-1}\right).$$

   c) $h(x) : \mathbb{R} \to \mathbb{R}$,
   $$h(x) = \arcsin\left(\frac{2x}{x^2+1}\right).$$