Class worksheet 6: Mathematical analysis 1

April 10, 2024

Name: _____

This is just to practice, no points are awarded. $\mathbb{N} = \{1, 2, \dots\}$, log with unspecified base is the natural logarithm.

1. Let $f\colon \mathbb{R} \to [0, 1]$ be continuous and satisfy $f(0) = f(1) = 0$. Does the graph of f have to intersect the straight line connecting the points $(0, 1)$ and $(1, -1)$?

2. Compute the function limits

   (a) $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

   (b) $\lim_{x\to\infty} \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}$

   (c) $\lim_{x\to\infty} \frac{\log(1+2^x)}{x}$

   (d) $\lim_{x\to 0} \frac{\sin 2x}{x}$

   (e) (*) $\lim_{x\to 0} \frac{\sin(\sin(\sin x))}{\tan(\tan x)}$

   (f) (*) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

   (g) (*) $\lim_{x\to e} \frac{\log x - 1}{x - e}$

   (h) (*) $\lim_{x\to 1} \frac{x+x^2+\cdots+x^n-n}{x-1}$, where $n \in \mathbb{N}$

3. Suppose the air temperature yesterday at midnight was 5 degrees Celsius, and today at midnight it was 5 degrees again.

   (a) Prove that there were two points in time exactly 12 hours apart that had equal temperature.

   (b) Is the above still true with 12 replaced by 8?

   (c) (*) What about $\sqrt{2}$ hours apart?