Class worksheet 5: Mathematical analysis 1

March 27, 2024

Name: _____

This is just to practice, no points are awarded.

1. Prove that $n! > n^{n/2}$. **Hint:** use the same method as for the computation of $\sum_{k=1}^{n} k$.

2. Prove that for every $x \in \mathbb{R}$ the series

$$\exp(x) := \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

   converges absolutely.

3. Show that $\exp(x+y) = \exp(x)\exp(y)$. You may treat infinite sums as if they were finite.

4. For $n \geq 2$ let $x(n)$ and $y(n)$ be the smallest and largest prime divisors of n, respectively. For the sequences below, find all cluster points, limes inferior and limes superior.

   (a) $a_n = \frac{1}{x(n)}$

   (b)* $b_n = \frac{y(n)}{x(n)}$