Class worksheet 4: Mathematical analysis 1

March 20, 2024

Name: _____

This is just to practice, no points are awarded. $\mathbb{N} = \{1, 2, \dots\}$, log is the natural logarithm.

1. Does the series converge, and why?

   (a) $\sum_{n=1}^{\infty} \frac{1}{2n+1}$

   (b) $\sum_{n=1}^{\infty} \frac{2n+5}{n^3+1}$

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

2. Compute the limit of the series

   (a) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$

   (b) $\sum_{n=1}^{\infty} \frac{2^{n+1}+3^n}{6^n}$

   (c) $\sum_{n=1}^{\infty} \frac{2n-1}{2^n}$

3. Prove that the series converges

   (a) $\sum_{n=1}^{\infty} \frac{1}{n!}$

   (b) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

   (c) $\sum_{n=1}^{\infty} \binom{2n}{n} \frac{1}{5^n}$

   (d) $\sum_{n=1}^{\infty} \left(\frac{n+1}{3n+2}\right)^n$

   (e) $\sum_{n=1}^{\infty} (\sqrt[n]{n} - 1)^n$

   (f) $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n - \log n}$