Class worksheet 3: Mathematical analysis 1

March 13, 2024

Name: ______________________________

This is just to practice. No points are awarded. $\mathbb{N} = \{1, 2, 3, \ldots\}$

1. Let (a_n) be a sequence such that $\lim_{n\to\infty} a_n = 1$. Decide whether the following statements about (a_n) are true (and justify your answer).
   (a) $\forall \varepsilon > 0\ \exists n_0\ \forall n > n_0\ |a_n - 1| < \varepsilon$
   (b) $\exists n_0\ \forall \varepsilon > 0\ \forall n > n_0\ |a_n - 1| < \varepsilon$
2. Compute the limits (if they exist) of the following sequences, as $n \in \mathbb{N}$ tends to infinity.
   (a) $\frac{\lfloor \sqrt{n} \rfloor}{\sqrt{n}}$
   (b) $\frac{3^n + 5^n + 7^n}{3^{n+1} + 5^{n+1} + 7^{n+1}}$
   (c) $\frac{(2n+1)^{20}(-3n+2)^{30}}{(4n-5)^{50}}$
   (d) $\frac{n^n}{n!}$
   (e) $\frac{2n^2 + 4n + n\sin n}{n\cos n + (2n + \sin n)^2}$
   (f) $\cos(n^2\pi) + \cos((n+1)\pi)$
   (g) $\sum_{k=1}^{n} (\frac{1}{2^k} + \frac{1}{3^k})$
   (h) $\sum_{k=1}^{n} \frac{1}{k(k+1)}$
3. Find two *different* sequences (a_n) and (b_n) such that (a_n) is a subsequence of (b_n) and vice versa.
4. (*) Determine, if the recurrently defined sequence has a limit. What is the limit?
   (a) $a_1 = 0$ and $a_{n+1} = a_n + \frac{1}{2}(x - a_n)^2$ for $0 \le x \le 1$
   (b) $a_1 = c$ and $a_{n+1} = \frac{1}{2}(a_n + \frac{2}{a_n})$, for $c \in (0, \infty)$