# Class worksheet 2: Mathematical analysis 1

March 6, 2024

Name: ______________________________

This is just to practice. No points are awarded. $\mathbb{N} = \{1, 2, 3, \ldots\}$

1. Does the sequence have a limit? What is the limit? Justify your answer.

   (a) $\frac{n-1}{n+1}$

   (b) $(3 + \frac{1}{n} + \frac{1}{n^2-1})(2 - \frac{1}{n^2})$

   (c) $\frac{n^3-10}{1-10n^3}$

   (d) $\frac{1+\ldots+n}{n^2}$

   (e) $\frac{\sqrt{n}}{n^2+1}$

   (f) $\frac{1}{n}\sin n^2$

   (g) $\cos(\frac{\pi n}{4})$

   (h) $\sqrt{n+5} - \sqrt{n-1}$

2. Let $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ be sequences in $\mathbb{R}$. Decide if true or false:

   (a) $\lim_{n\to\infty} a_n = a \Leftrightarrow \lim_{n\to\infty} a_{n+1} = a$

   (b) $\lim_{n\to\infty} a_n = a \Leftrightarrow \lim_{n\to\infty} a_{2n} = a$

   (c) $(\lim_{n\to\infty} a_n = a, \lim_{n\to\infty} b_n = b$ and $\forall n \in \mathbb{N}\colon\ a_n \leq b_n) \Rightarrow a \leq b$.

   (d) $(\lim_{n\to\infty} a_n = a, \lim_{n\to\infty} b_n = b$ and $\forall n \in \mathbb{N}\colon\ a_n < b_n) \Rightarrow a < b$.

3. (*) Prove that for any set $X$ we have $|\mathcal{P}(X)| > |X|$. **Hint:** Suppose that $f : X \to \mathcal{P}(X)$ is a bijection. Consider $Y = \{x \in X : x \notin f(x)\}$.

4. (*) (Cantor-Bernstein theorem) Let $X$ and $Y$ be two (infinite) sets such that there exist injections $f : X \to Y$ and $g : Y \to X$. Show that $|X| = |Y|$ (i.e., construct a bijection between $X$ and $Y$).