Class worksheet 1: Mathematical analysis 1

28 Feb 2024

Name: _____

This is just to practice. No points are awarded.

1. Is this set countable?
   (a) $\mathbb{N}$
   (b) $\mathbb{Z}$
   (c) $\mathbb{Q}$
   (d) $\mathbb{R}$
2. Is this set countable?
   (a) $\mathbb{Z} \times \mathbb{Z}$
   (b) $\mathbb{Q} \times \mathbb{Q}$
   (c) $\mathbb{N}^3$
   (d) $\mathbb{R} \setminus \mathbb{Q}$
   (e) $\mathcal{P}(\mathbb{N})$
3. Determine the supremum and infimum in $\mathbb{R}$ of each of the following sets. Is this value also the maximum/minimum?
   (a) $\{1/n\colon\ n \in \mathbb{N}\}$
   (b) $\{z \in \mathbb{Q}\colon\ z^2 < 3\}$
   (c) $\{\sin x\colon\ x \in [0, 2\pi)\}$
   (d) $\{\sin x\colon\ x \in (0, \pi)\}$
   (e) $\{\frac{z-1}{z} :\ z \in \mathbb{Z} \setminus \{0\}\}$
   (f) $\{\frac{m}{m+n}\colon\ m, n \in \mathbb{N}\}$
4. Negate the statement. Decide if the statement or its negation is true.

$$\forall x_1 \in \mathbb{R}\ \exists y_1 \in \mathbb{R}\ \forall x_2 \in \mathbb{R}\ \exists y_2 \in \mathbb{R}\ \forall x_3 \in \mathbb{R}\ \exists y_3 \in \mathbb{R}\colon\ x_1 < y_2 < x_3 \Rightarrow y_1 < x_2 < y_3.$$