3rd homework set for Mathematical analysis
Due: 15 May, 2024 at 10:40 AM. Total homework points are capped at 40.

---

1. *[3 points]* With the aid of Taylor polynomials, compute the following limits.

   a) $\lim_{x\to 0} \dfrac{e^x \sin x - x(1+x)}{x^3}$.

   b) $\lim_{x\to\infty} \Big((x^6+x^5)^{1/6} - (x^6-x^5)^{1/6}\Big)$.

   c) $\lim_{x\to\infty} (x - x^2 \log(1+\dfrac{1}{x}))$.

2. *[7 points]* Let $f : \mathbb{R} \to \mathbb{R}$ be a function.

   a) Suppose $f'(0) = f''(0) = 0$ and $f^{(3)}(0) > 0$. Is 0 a local minimum, maximum or neither?

   b) Suppose $f'(0) = f''(0) = f^{(3)}(0) = 0$ and $f^{(4)}(0) > 0$. Is 0 a local minimum, maximum or neither?

   c) Suppose $f^{(k)}(0) = 0$ for all $k \in \mathbb{N}$. Can f be strictly increasing on the interval $(-\epsilon, \epsilon)$, for some $\epsilon > 0$?

3. *[6 points]* Using substitution, convert the integrals below to integrals of a rational function. You do not have to calculate the resulting integrals.

   a) $\displaystyle\int \frac{1}{\sin x \cos x}\, dx$.

   b) $\displaystyle\int \frac{1}{\sin x}\, dx$.

   c) $\displaystyle\int (\tan x)^5\, dx$.

4. *[4 points]* Compute the area

   a) Between the parabola $y = x^2$ and the x-axis

   b) Below the $y = \sin x$ curve on $[0, \pi]$

   c) Between the graphs of $\sin x$ and $\cos x$ on $[0, 2\pi]$

   d) Of the unit circle