2nd homework set for Mathematical analysis
Due: April 24, 2024 at 10:40 AM.

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1. *[6 points]* Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined via $f(x) = x^{3+\operatorname{sgn} x}$ where

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{for } \; x > 0 \\ 0 & \text{for } \; x = 0 \\ -1 & \text{for } \; x < 0 \end{cases}$$

   Determine the (two-sided) derivative of f at every point where it exists.

2. *[4 points]* We cut out small squares from the corners of a square sheet of paper and fold it to make a box (without a lid). How large should the cut-out squares be to maximize the volume of the box that is formed?

3. *[6 points]* Using the L'Hospital rule, compute the limits

   (a) $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\sin 2x}{x}$

   (c) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x^2}$

   (d) $\displaystyle\lim_{x\to 0} \frac{\sin(\sin(\sin x))}{\tan(\tan x)}$

   (e) $\displaystyle\lim_{x\to e} \frac{\log x - 1}{x - e}$

   (f) $\displaystyle\lim_{x\to 1} \frac{x + x^2 + \cdots + x^n - n}{x - 1}$, where $n \in \mathbb{N}$

4. *[4 points]* Using Taylor polynomials, numerically approximate the following values up to six digits after the decimal point. You can omit error estimates.

   a) $\exp(0.01)$

   b) $\cos 0.1$

   c) $\sqrt{0.98}$

   d) $\log 1.2$