Home assignment 0

Mathematical Analysis 1

Submission deadline: 6 March, 10:40

This is a **test** for the submission system and an opportunity for you to practice. No points are awarded, submission is voluntary.

1. Prove that $\sqrt{3} \notin \mathbb{Q}$ and $\log_2 3 \notin \mathbb{Q}$.

2. Negate the statement. Decide if the statement or its negation is true.

$$\forall x_1 \in \mathbb{R}\ \exists y_1 \in \mathbb{R}\ \forall x_2 \in \mathbb{R}\ \exists y_2 \in \mathbb{R}\ \forall x_3 \in \mathbb{R}\ \exists y_3 \in \mathbb{R}\colon\ x_1 < y_2 < x_3 \Rightarrow y_1 < x_2 < y_3.$$

3. Give an *algebraic form* of an injective function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$. That is, give a formula for the value of $f(i, j)$ for an arbitrary (i, j).

4. Prove that the set of all *finite* subsets of $\mathbb{N}$ is countable.

5. Prove that $(0, 1)$ and $\mathbb{R}$ have the same cardinality. **Hint:** use a trigonometric function. Deduce that a countable union of sets, each of cardinality of $\mathbb{R}$, also has cardinality of $\mathbb{R}$.

6 (*). Prove that $\mathbb{R} \times \mathbb{R}$ and $\mathbb{R}$ are of the same cardinality.