Home assignment 4

Combinatorics and Graphs 1

Submission deadline: 13 December, 12:20

Give rigorous proofs to your claims. Facts from the lecture can be used without a proof.

1. Let G be a connected graph with at least one edge, and in which all vertices have even degree.

(a) Must G be 2-connected?

(b) Must G be 2-edge-connected?

2. Let $k \geq 2$ be an integer. Prove that if a connected bipartite graph is k-regular then it is 2-connected. What about k-regular non-bipartite graphs?

3. Let $\mathcal{P}$ be a finite projective plane and $G = (V, E)$ be its incidence graph.

(a) Prove that G is C_4-free.

(b) Prove that $|E| = \Theta(|V|^{3/2})$.

4. Prove that any n-vertex graph that does not contain the complete bipartite graph $K_{2,t}$ as a subgraph has at most $\frac{1}{2}n^{3/2}\sqrt{t-1}+o(n^{3/2})$ edges. **Hint:** Imitate the proof from Lecture 9 for $C_4 = K_{2,2}$.