Home assignment 2

Combinatorics and Graphs 1

Submission deadline: 8 November, 12:20

Give rigorous proofs to your claims. Statements proved in the lecture can be used without a proof. $[n]$ stands for $\{1, \ldots, n\}$.

1. Prove assertion (iii) of Theorem 1 from the lecture, aka Theorem 3.1.4 of the notes, *directly*, that is, without invoking duality (Theorem 3.2.2).

2. Let $n \geq 4$ be an integer, and let $X = \{0, 1, \ldots, n\}$. Construct a set $\mathcal{F} \subseteq 2^X$ such that

- $(X, \mathcal{F})$ satisfies (P1) and (P2) from the definition of a finite projective plane,
- $(X, \mathcal{F})$ does **not** satisfy (P0) from the definition of a finite projective plane, and
- Every $L \in \mathcal{F}$ satisfies $|L| \geq 2$, and there are at least two lines.

3. Assuming, there exist two orthogonal Latin squares of order n, prove that the numbers $1, 2, \ldots, n^2$ can be arranged in an $n \times n$-table such that all row sums and column sums are the same.

4. Let L be the Latin square of order $2n$, where $n \in \mathbb{N}$, defined by $L(i, j) = i + j \mod 2n$ (evaluated to be integer between 1 and $2n$). Prove that there is no Latin square orthogonal to L. **Hint:** Show that, in fact, L does not have a single transversal.