

Home assignment 0

Combinatorics and Graphs 1

Submission deadline: 11 October, 12:20

This is a **test** for the submission system and an opportunity for you to practice. No points are awarded, submission is voluntary.

1. Prove that for $k = o(\sqrt{n})$ we have $\binom{n}{k} = (1 + o(1)) \frac{n^k}{k!}$.
2. Show that the number of subsets of $[n] := \{1, \dots, n\}$ of even cardinality is 2^{n-1} . If n is divisible by 8, what is the number of subsets of $[n]$ of cardinality divisible by 4? (*Hint*: consider $(1 + i)^n$, where $i = \sqrt{-1}$ is the imaginary unit.)
3. Let A_1, \dots, A_n be finite sets. Recall the principle of inclusion-exclusion used to express $|\bigcup A_i|$, and the proof thereof. Show that if you take into account only the first $m < n$ sums in the formula, you will get an overestimate when m is odd and an underestimate when m is even.