Class worksheet 8: Combinatorics and Graphs 1

December 6, 2023

Name: ____

This is just to practice, no points are awarded.

1. (a) Let T be the tree on $[9]$ with the edges $12, 23, 34, 45, 46, 47, 28, 29$. Find the Prüfer code of T.

   (b) Find the tree on $[7]$ whose Prüfer code is $3, 3, 7, 4, 3$.

2. Find the smallest $n \in \mathbb{N}$ such that, for every graph G on at least n vertices, either G contains a triangle or $\overline{G}$ (the complement of G) contains $K_{1,3}$ as a subgraph.

3. Let k be a positive integer. Prove that there exists N such that any sequence $a_1, \dots, a_N$ of distinct real numbers contains a subsequence of length k that is either strictly increasing or strictly decreasing.

4. (a) Prove Ramsey's theorem for any number of $k \geq 2$ colours. That is, show that for any $t_1, \dots, t_k \geq 2$ there exists n such that in every k-colouring of $E(K_n)$ there must be a monochromatic t_i-clique of colour i, for some $i \in [k]$.

   (b) Prove for every k there exists N such that in any (vertex-)colouring $\phi : [N] \to k$ there exist $x, y, z \in [N]$ that $\phi(x) = \phi(y) = \phi(z)$ and $x + y = z$. **Hint:** define a k-colouring ψ of $\binom{[N]}{2}$ via $\psi(\{x, y\}) = \phi(|x - y|)$. Now apply Ramsey's theorem.