

Class worksheet 7: Combinatorics and Graphs 1

November 22, 2023

Name: _____

This is just to practice, no points are awarded.

1. Prove Hall/König by applying (local) Menger to an appropriate auxiliary graph.
2. Let G be a connected graph with at least one edge, and in which all vertices have even degree.
 - (a) Must G be 2-connected?
 - (b) Must G be 2-edge-connected?
3. Let $k \geq 2$ be an integer. Prove that if a connected bipartite graph is k -regular then it is 2-connected. What about k -regular non-bipartite graphs?
4. Let k and ℓ be integers with $1 \leq k < \ell$. Construct graphs F , G and H such that ¹
 - (a) $\kappa(F) = k$ and $\kappa(F - x) = \ell$ for some vertex $x \in V(F)$.
 - (b) $\lambda(G - x) = k$ and $\lambda(G - xy) = \ell$ for some edge $xy \in E(G)$.
 - (c) $\kappa(H) = k$ and $\lambda(H) = \ell$.

¹ $G - x$ stands for “ G with vertex x and the incident edges removed”. $G - xy$ stands for “ G with edge $xy \in E(G)$ removed but vertices x and y retained”