Class worksheet 3: Combinatorics and Graphs 1

October 25, 2023

Name: _____

This is just to practice, no points are awarded. $\mathbb{N} = \{1, 2, \dots\}$, log with unspecified base is the natural logarithm.

1. Using just the axioms prove that there exists no projective plane of order 1.

2. Let $(X, \mathcal{F})$ be a finite projective plane and $(Y, \mathcal{G}) = (X, \mathcal{F})^*$ be its dual. Prove that $(Y, \mathcal{G})$ satisfies (P2) (we skipped this proof in the lecture).

3. Prove assertion (iii) of Theorem 1 from the lecture, aka Theorem 3.1.4 of the notes, *directly*, that is, without invoking duality.

4. (*) (Kirkman's schoolgirl problem, 1850, with $n = 9$ instead of 15 originally) Nine young ladies in a school walk out three abreast for four days in succession: it is required to arrange them daily so that no two shall walk twice abreast.

5. (*) Construct a finite projective plane of order 3.