

Class worksheet 2: Combinatorics and Graphs 1

October 11, 2023

Name: _____

This is just to practice, no points are awarded. $\mathbb{N} = \{1, 2, \dots\}$, log with unspecified base is the natural logarithm.

1. Use generating functions to find a closed formula for the following recursively defined sequences.
 - (a) $z_0 = 1; z_{n+1} = z_n + 1$ for all $n \geq 0$.
 - (b) $a_0 = 1; a_{n+1} = 2a_n + 3$ for all $n \geq 0$.
 - (c) $b_0 = b_1 = 1; b_{n+2} = b_{n+1} + 6b_n$ for all $n \geq 0$.
2. In how many ways can you tile a $2 \times n$ board by 1×2 and 2×2 tiles? You may rotate the 1×2 tile.
3. A *bracket expression* is a string of opening and closing brackets. The expression is *regular* if it has as many opening as closing brackets in total, and every prefix has at least as many opening brackets as closing ones. Let c_n be the number of regular bracket expressions of length $2n$. Find a recurrence relation for the sequence $(c_n)_{n=0}^{\infty}$.
4. (*) In how many ways can you tile a $3 \times 2n$ board by 1×2 tiles?