1. Using dynamic programming over the subsets, obtain an algorithm for Chromatic Number on n-vertex graphs running in time $3^n n^{\mathcal{O}(1)}$.

2. Using dynamic programming over the subsets, obtain an algorithm for Hamiltonian Cycle on n-vertex graphs running in time $2^n n^{\mathcal{O}(1)}$.

3. Show an algorithm which computes the number of perfect matchings in a given n-vertex bipartite graph in $2^{n/2} n^{\mathcal{O}(1)}$ time and polynomial space.

4. In the Connected Vertex Cover problem, we are given a graph G and an integer $k \in \mathbb{N}$. The goal is to output a subset of vertices $S \subseteq V(G)$ such that S is a vertex cover of G, S is connected, and $|S| \leq k$.

   Show that Connected Vertex Cover admits an algorithm with running time $6^k n^{\mathcal{O}(1)}$.

   Hint. You might need to use an algorithm for Steiner Tree from the lecture.