1. A *feedback vertex set* Z of graph G is a subset of vertices such that $G - Z$ is a forest.

   Show that if a graph on n vertices has minimum degree at least 3, then it contains a cycle of length at most $2\lceil \log n \rceil$. Use this to design a $(\log n)^{\mathcal{O}(k)} n^{\mathcal{O}(1)}$-time algorithm for Feedback Vertex Set on undirected graphs. Is this an FPT algorithm for Feedback Vertex Set?

2. In the Min-Ones-r-SAT problem, we are given an r-CNF formula φ and an integer k. The objective is to decide whether there exists a satisfying assignment for φ with at most k variables set to true. Show that Min-Ones-r-SAT admits an algorithm with running time $f(r,k)n^{\mathcal{O}(1)}$ for some computable function f.

3. Describe an algorithm running in time $\mathcal{O}(1.5^n)$ which finds the number of independent sets (or, equivalently, vertex covers) in a given n-vertex graph.

   You may need to prove that counting the number of independent sets in graphs of degree at most 2 is polynomial time solvable.

4. Let $\mathcal{F}$ be a set of graphs. We say that a graph G is $\mathcal{F}$-free if G does not contain any induced subgraph isomorphic to a graph in $\mathcal{F}$; in this context the elements of $\mathcal{F}$ are sometimes called forbidden induced subgraphs. For a fixed set $\mathcal{F}$, consider a problem where, given a graph G and an integer k, we ask to turn G into a $\mathcal{F}$-free graph by:

   (vertex deletion) deleting at most k vertices;

   (edge deletion) deleting at most k edges;

   (completion) adding at most k edges;

   (edition) performing at most k editions, where every edition is adding or deleting one edge.

   Considering $\mathcal{F}$ to be a fixed set means that $|\mathcal{F}| \in \mathcal{O}(1)$ and every graph in $\mathcal{F}$ has size $\mathcal{O}(1)$.

   Prove that, if $\mathcal{F}$ is finite, then there exists a $2^{\mathcal{O}(k)} n^{\mathcal{O}(1)}$-time FPT algorithm for each of the four aforementioned problems. (Note that the constants hidden in the $\mathcal{O}()$-notation may depend on the set $\mathcal{F}$.)