Tutorial 2

power set

Problem 1.

1.) if $x \in A \cup (B \cap C)$, then $x \in A$ or ($x \in B$ and $x \in C$)

- if $x \in A$ then $x \in A \cup B$ and $x \in A \cup C$,

  so $x \in (A \cup B) \cap (A \cup C)$
- if $x \in B$ then $x \in A \cup B$

  $x \in C$ then $x \in A \cup C$

  so if $x \in B$ and $x \in C$, then $x \in (A \cup B) \cap (A \cup C)$

so $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

2.) if $x \in (A \cup B) \cap (A \cup C)$, then ($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

so $x \in A$ or ($x \in B$ a $x \in C$) so $x \in A \cup (B \cap C)$

Problem 2

a) $\sup M = 0$, $\inf = -1$, $= \min$, non max

argument similar to one at the lecture

b) $\inf = 0$ (contains $\frac{1}{n}$ for $n > 1$), not min

$\sup = 1$ (contains $1 - \frac{q}{p+q}$), not max

c) $\inf = 0$ (for $n = 1$) $\min = 0$

$\sup = \frac{3}{2}$ (for $n = 2$) $\max = \frac{3}{2}$

need to show that $\frac{n + (-1)^n}{n} \leq \frac{3}{2}$ for every n

(separately for odd and even: odd: left side < 1 ✓

even $\Leftrightarrow 2(n+1) \leq 3n \Leftrightarrow 2 \leq n$ ✓

d) $\sup = \infty$ for every k, there is $n^{(-1)^n} > k$ (e.g. $2\lceil k \rceil$)

$\inf = 0$ $\frac{1}{2k+1} \in M$ $\forall k \in \mathbb{N}$, $\forall \varepsilon \; \exists k: \frac{1}{2k+1} < \varepsilon$

no max/min

e) $\sup = \frac{1}{2} + \frac{1}{3} = \max$

$\inf = 0$ not min

f) $\sup = \infty$

$\inf = 0$

g) $\sup = \infty$

$\inf = 0$

Problem 3 $\inf = \{5, 9\}$

$\sup = \{1, 2, 3, 4, 5, 6, 7, 9, 10, 11\}$

Problem 4 a) ~~max~~ $\inf (A \cup B) = \min(\inf A, \inf B)$

$\sup (A \cup B) = \max(\sup A, \sup B)$

b) $\sup \leq \min(\cancel{\sup}\ \sup A, \sup B)$

$\inf \geq \max(\inf A, \inf B)$

c) $\sup, \inf \in \cancel{\sup} (\inf A, \sup A)$