Tutorial 1

Problem 1

a) False — 9 not prime OR 9 ~~not~~ even

b) True — 9 not prime AND 9 even

c) True — 9 prime AND 9 even

d) False — (9 prime AND 9 even) OR (9 not prime AND & 9 odd)

Problem 2

1. TRUE $(y \leq x)$ $\exists x \; \forall y \; \exists z : (z > x) \wedge (z \leq y)$

2. FALSE $\forall y \; \exists x \; \exists z : (z > x) \wedge (z \leq y)$
(TRUE for $\mathbb{N}$ and $y = 1$)

Problem 3

neg. : $\exists \varepsilon > 0 \; \forall k > 0 \; \exists x : (x > k) \wedge (|f(x)| \geq \varepsilon)$

• False, neg. true $(\varepsilon = 1/2, \; x = \frac{\pi}{2} + 2k\pi)$

Problem 4

$B \Rightarrow A$

Problem 5

2 cases: $y \geq 0$ so $|y| = y$; $y < 0$ so $|y| = -y$

„$\Rightarrow$" $y \geq 0$: $((y \leq x) \wedge (-x \leq y)) \Rightarrow y \leq x$

$y < 0$: $(y \leq x \wedge -x \leq y) \Rightarrow -y \leq x$

„$\Leftarrow$"

$y \geq 0$: $y \leq x \Rightarrow y \leq x$

$-x \leq 0 \leq y$ ✓

$-x \leq y \quad / \cdot (-1)$

$x \geq -y$ or $-y$, $+x$

$y < 0$: $-y \leq x \Rightarrow -x \leq y$

$y < 0 < x$

$\blacksquare$

Problem 6

$n = 1 \qquad 2 - 1 = 1^2$

$$LS = \sum_{i=1}^{n+1} (2i-1) = \sum^{n} + 2(n+1) - 1 \overset{IH}{=} n^2 + 2(n+1) - 1$$

$$= n^2 + 2n + 1 = (n+1)^2 = RS$$

Problem 7

a) $\frac{x_1+x_2}{2} \geq \sqrt{x_1 x_2}$

- $(\sqrt{x_1}+\sqrt{x_2})^2 \geq 0$

$x_1 + 2\sqrt{x_1 x_2} + x_2 \geq 0$

$x_1 + x_2 \geq 2\sqrt{x_1 x_2}$

$\frac{x_1+x_2}{2} \geq \sqrt{x_1 x_2}$

b) $k = 1$ a)

1.step: $LS = \frac{x_1 + \dots + x_{2^{k+1}}}{2^{k+1}} = \frac{x_1 + \dots + x_m}{2m} + \frac{x_{m+1} + \dots + x_{2m}}{2m}$

$m = 2^k$

$2^{k+1} = 2m$

IH for m

goal for $2m$

$\overset{IH}{\geq} \frac{\sqrt[m]{x_1 \dots x_m} + \sqrt[m]{x_{m+1} \dots x_{2m}}}{2}$

$\overset{a)}{\geq} \sqrt{\sqrt[m]{x_1 \dots x_m} \cdot \sqrt[m]{x_{m+1} \dots x_{2m}}} = \sqrt[2m]{x_1 \dots x_{2m}}$

Problem 8

- $2 | 2$ ✓
- I. hyp: $\forall k = 1 \dots n$ div. by a prime
  - $n+1$
    - div. only by $n+1$ and $1 \to$ prime ✓
    - has some other divisor d, $d < n+1$
      - d a prime ✓
      - in any case, by IH d is divisible by some prime p

Thus $d = k_1 \cdot p$ for some $k_1 \in \mathbb{Z}$

$n+1 = d \cdot k_2$ for some $k_2 \in \mathbb{Z}$

$n+1 = p \cdot (k_1 \cdot k_2) \to$ divisible by p ∎

Problem 9: Let $p_1 \dots p_k$ be a set of all primes.
Consider $(p_1 \cdot p_2 \cdot \dots \cdot p_k) + 1$. It is not divisible by $p_1 \dots p_k$.
But by Prob. 8 it is div. by some prime $\not{\!\!\!\downarrow}$