

Mathematical analysis I — Replacement homework

Due: at least 5 days before you want to attend the exam.

Mark from submitted homework will replace your lowest score (if higher) from the respective homeworks.

Problem 1: Replacement for homework 1-4. Show that $\lim_{n \rightarrow \infty} \sin(n)$ does not exist.

Problem 2: Replacement for homework 5-8. Let $\sum_{n=0}^{\infty} a_n$ be an absolutely convergent series and let $f : \mathbb{N} \rightarrow \mathbb{N}$ be a strictly increasing function. Prove that the series $\sum_{n=0}^{\infty} a_{f(n)}$ is absolutely convergent. Show that the statement is not true if we replace "absolutely convergent" by "convergent".

Problem 3: Replacement for homework 9-12. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be continuous functions such that $f(x) = g(x)$ for every $x \in \mathbb{Q}$. Using the definition of continuity, show that $f(x) = g(x)$ for all real numbers. (Hint: for $c \in \mathbb{R} \setminus \mathbb{Q}$ write the definition of continuity of f and g at c and using triangle inequality, show that for every $\varepsilon > 0$, $|f(c) - g(c)| < 2\varepsilon$.)