Mathematical analysis I — Homework 9

Due: 15:40, 5.12.

Write your solution of each problem on a separate sheet of paper. One part will be marked for credit.

Problem 1: Using comparison test, decide whether following series converege or diverge:

(a) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{3^n+7^n}{3^n+8^n}$

(c) $\sum_{n=1}^{\infty} \frac{\sqrt{n}+2}{n^{3/2}+1}$

Problem 2: Decide whether following series converege or diverge:

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$

(b) $\sum_{n=1}^{\infty} \left(\frac{2}{3}+\frac{1}{n}\right)^n$

(c) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Problem 3:

(a) Give examples of series demonstrating that the comparison criterion does not work when terms of series can be negative.

(b) Prove that if for all positive integers n $a_n > 0$, $a_n \le b_n \le c_n$, $\sum_{n=1}^{\infty} a_n = 7$ and $\sum_{n=1}^{\infty} c_n = 10$, then $\sum_{n=1}^{\infty} b_n$ is convergent. (Bear in mind that terms of the series can be negative, therefore the comparison criterion does not apply.)

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