Mathematical analysis I — Homework 6

Due: 15:40, 14.11.

Write your solution of each problem on a separate sheet of paper. One part will be marked for credit.

Problem 1: Calculate the following limits.

(a) $\lim_{n\to\infty} \frac{n^4 11^n + n^9 9^n}{7^{2n}+1}$

(b) $lim_{n\to\infty} \frac{2n^2+4n+n\sin n}{n\cos 3n+(2n+\sin n)^2}$

Problem 2: Calculate the following limits.

(a) $\lim_{n\to\infty} \sqrt[n]{3n^2+n}$

(b) $\lim_{n\to\infty} \sqrt[n]{2^n-n}$

Problem 3: Let $(a_n)_{n=0}^{\infty}$ be a sequence if positive reals such that for some $0 < r < 1$ there is $n_0 \in \mathbb{N}$ such that every $n \geq n_0$ satisfies $a_{n+1}/a_n \leq r$. Prove that then $\lim_{n\to\infty} a_n = 0$. (Hint: First prove by induction that $a_n \leq r^n \cdot a_{n_0}$.) Use this to show that $n!$ grows much faster than 10^n, in other words, $\lim_{n\to\infty} \frac{10^n}{n!} = 0$.

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