

## Mathematical analysis I — Homework 5

**Due: 15:40, 7.11.**

Write your solution of each problem on a separate sheet of paper. One part will be marked for credit.

*Problem 1:* Using arithmetic of limits, calculate the following limits. You can use the statement from Problem 3 as a fact.

(a)  $\lim_{n \rightarrow \infty} \frac{3n^2+n}{-n^2+4n}$

(b)  $\lim_{n \rightarrow \infty} \frac{\sqrt{3x^2+6}}{5-2x}$

(c)  $\lim_{n \rightarrow \infty} \sqrt{n+5} - \sqrt{n-1}$

*Problem 2:* Compute the limit of the sequence  $(a_n)$ , where  $a_n = \sin(\frac{3}{5}\pi n)$  or prove it does not exist. (Argument of sin is in radians, that is,  $\sin \frac{\pi}{2} = 1$ ,  $\sin 2\pi = 0$ ).

*Problem 3:* Prove that if a sequence of non-negative reals  $(a_n)$  has limit  $a \in \mathbb{R}$  (note that  $a$  must be non-negative), then  $\lim_{n \rightarrow \infty} \sqrt{a_n} = \sqrt{a}$ .

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