# Mathematical analysis I — Tutorial 4

http://kam.mff.cuni.cz/∼tereza/teaching.html

*Problem 1:* Decide whether the following sequences are monotone. If yes, are they increasing, decreasing, non-increasing or non-decreasing?

a) $(2^{-n})_{n=1}^{\infty}$

b) $(2n + (-1)^n)_{n=1}^{\infty}$

c) $(\sin n)_{n=1}^{\infty}$

d) $\left(\dfrac{1}{1+n^2}\right)_{n=1}^{\infty}$

e) $\left(\dfrac{n+1}{n+2}\right)_{n=1}^{\infty}$

f) $(\sqrt{n+1} - \sqrt{n})_{n=1}^{\infty}$

*Problem 2:* A sequence $(a_n)$ is known to be increasing.

a) Might it have an upper bound?

b) Might it have a lower bound?

c) Must it have an upper bound?

d) Must is have a lower bound?

Give a numerical example to illustrate each possibility or impossibility.

*Problem 3:* If a sequence is not bounded above, must it contain

a) a positive term,

b) an infinite number of positive terms?

*Problem 4:* Think of examples to show that:

a) an increasing sequence need not tend to infinity;

b) a sequence that tends to infinity need not be increasing;

c) a sequence with no upper bound need not tend to infinity.

*Problem 5:* Justify that the following properties do not imply that a sequence tends to zero (i.e. its limit is zero).

a) A sequence in which each term is strictly less than its predecessor.

b) A sequence in which each term is strictly less than its predecessor while remaining positive.

c) A sequence in which, for sufficiently large n, each term is less than some small positive number.

d) A sequence with arbitrarily small terms.

Use the following sequences for your arguments.

$$a_n = 3 - n, \qquad b_n = \frac{n+1}{n}, \qquad c_n = -\frac{1}{10}, \qquad d_n = \begin{cases} 1 \text{ for } n \text{ odd}, \\ \frac{1}{2^n} \text{ for } n \text{ even} \end{cases}$$

*Problem 6:* For the sequence $a_n = 1 + \frac{1}{\sqrt{n}}$, find $n_0$ such that for every $n \geq n_0$

a) $|a_n - 1| < 0,1$

a) $|a_n - 1| < 0,01$

*Problem 7:* Prove that the sequence $a_n = \frac{(-1)^n}{n}$ does not converge to 2.

*Problem 8:* Using the definition, find (and prove) limits of the following sequences:

a) $\left(\frac{1}{n}\right)_{n=1}^{\infty}$

b) $(\log n)_{n=1}^{\infty}$

c) $\left(\frac{1}{1+n^2}\right)_{n=1}^{\infty}$

d) $\lim_{n\to\infty} (-1)^n$

e) $\lim_{n\to\infty} \cos(-1)^n$

f) $\lim_{n\to\infty} (-1)^{n!}$

g) $\lim_{n\to\infty} \frac{(-1)^n}{n}$

h) $\lim_{n\to\infty} \cos\left(\frac{n\pi}{4}\right)$

i) $\lim_{n\to\infty} \frac{2n+1}{3n-2}$

# Mathematical analysis I — Homework 4

**Due: 15:40, 31.10.**

Write your solution of each problem on a separate sheet of paper. One part will be marked for credit.

*Problem 1:* Find limits of the following sequences. For given $\varepsilon > 0$, find suitable $n_0$ from the definition of the limit.

a) $\left(\frac{2n}{n-1}\right)_{n=2}^{\infty}$

b) $\left(\sqrt{n+1}-\sqrt{n}\right)_{n=1}^{\infty}$

*Problem 2:* Decide and prove whether the sequence $(a_n)$ defined as $a_1 = 1$ and $a_{n+1} = \sqrt{a_n + 1}$ is bounded and monotone or eventually monotone. If it is monotone, show whether it is increasing, decreasing, non-increasing or non-decreasing and whether it is bounded. Conclude that the sequence must have a proper limit (you do not have to find it.)

*Problem 3:* Prove that a sequence $(a_n)$ cannot have a proper limit $a \in \mathbb{R}$ and an improper limit $\infty$ at the same time.