Mathematical analysis I — Homework 3

Due: 15:40, 23.10.

Write your solution of each problem on a separate sheet of paper. One part will be marked for credit.

Problem 1: Let $A = \{a \in \mathbb{R} : -3 < a \leq 1\}$ and $B = \{\in \mathbb{R} : -3 < b < 1\}$. Which statements are true?

(a) $\forall a \in A\ \exists b \in B\ b < a$

(b) $\exists b \in B\ \forall a \in A\ b < a$

(c) $\forall b \in B\ \exists a \in A\ b < a$

(d) $\exists a \in A\ \forall b \in B\ b < a$

Problem 2: Decide whether the function $f : \mathbb{N} \to \mathbb{N}$ defined as $f(x) = x(x+1)/2 - x + 1$ is injective and surjective. Prove it.

Problem 3: Prove *density* of rational and irrational numbers: show that for any who reals a, b such that $a < b$, there exist $r \in \mathbb{Q}$ and $s \in \mathbb{R} \setminus \mathbb{Q}$.

Hint: First part: First show that there is a natural number n such that $1/n < b - a$. Then argue, that there exists an integer m such that $a < m/n < b$. Second part: Pick your favourite irrational number j and find s in the form $j + p/q$, where p and q are integers, using the first part of the statement. Note that the sum of rational and irrational number is irrational.

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