Mathematical analysis I — Homework 2

Due: 15:40, 17.10.

Write your solution of each problem on a separate sheet of paper. One part will be marked for credit.

Problem 1: Determine suprema, infima, minima and maxima of the following sets in real numbers, if they exist. Justify your answers.

(a) $M_2 = \{2^{2^k} | k \in \mathbb{Z}\}$

(b) $M_3 = \{\frac{1}{1+\ln n} | n \in \mathbb{N}\}$

Problem 2: For any two sets A an B, prove that

(a) $A \subseteq B$ if and only if $A \cap B = A$

(b) $A \subseteq B$ if and only if $A \cup B = B$

Deduce from the previous two statements that $A \cap B = A$ if and only if $A \cup B = B$.

Problem 3: Define C as a set of all rational numbers q from interval $(0, 1)$ such that every digit (i.e., each of $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9) appears infinitely many times in the decimal expansion of q. E.g., $0, \overline{0123456789} \in C$, but $0, 012345678\overline{912} \notin C$. Also $\pi - 3 \notin C$, since it is irrational. (Recall that rational numbers have finite or periodic decimal expansion.) Decide whether C has supremum, infimum, minimum and maximum and find them if they exist.

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