Mathematical analysis I — Homework 10

Due: 15:40, 12.12.

Write your solution of each problem on a separate sheet of paper. One part will be marked for credit.

Problem 1: Decide whether following series converege (conditionally), converge absolutely or diverge:

(a) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{2|\cos \frac{n\pi}{2}|+(-1)^n n}{\sqrt{(n+1)^3}}$

Problem 2: For every $x \in \mathbb{R}$ decide, whether the following sequences are convergent (conditionally), absolutely convergent or divergent:

(a) $\sum_{n=1}^{\infty} \frac{(2x)^n}{n}$

(b) $\sum_{n=1}^{\infty} (-nx)^n$

Problem 3:

(a) Using Bernoulli's inequality show that the sequence $\left((1+\frac{1}{n})^n\right)$ is non-decreasing.

(b) Use Binomial Theorem to show that $(1+\frac{1}{n})^n \leq \sum_{k=0}^{n} \frac{1}{k!}$.

(c) Conclude that $\lim_{n\to\infty}(1+\frac{1}{n})^n$ exists and is at most e. (Use that $e = \sum_{k=0}^{\infty} \frac{1}{k!}$.)

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