Exam requirements for MAI055, 2019

Before taking an exam, it is necessary to obtain a pass from tutorials. The exam will be oral. Use of any electronics or written/printed material will not be allowed.
You will first get initial questions and some time to think about them and write notes about your thoughts, then we will discuss. I will ask additional questions and clarifications based on your performance.
The following three areas will be evaluated, passing grade in each is necessary to pass the exam, final mark will be determined as the average of the three.

1. Knowledge of the material from lectures and tutorials. Examples of questions:
   - Define a notion from the lecture. (e.g.: Define a limit of a sequence.)
   - State a theorem from the lecture (e.g.: State Leibniz criterion of convergence.)
   - Prove a theorem (that was proven during the lecture) (e.g.: Prove Bolzano-Weierstrass theorem.)
   - Write what you know about a certain topic. (e.g. Rearrangement of series.)
2. "Computational" applications - applying knowledge to solve problems. Emphasis will be given to correct justification of steps of calculations - which theorems are used, why and whether assumptions of the theorems are satisfied. Examples of questions:
   - Find limit of a sequence.
   - Determine whether a series is (absolutely) convergent.
   - Find a limit of a function at a point.
   - Find a derivative of a function.
   - Plot a graph of a function.
3. "Theoretical" applications. Examples of questions:
   - Decide wheter object with specified properties exists and construct an example of it (e.g. Sequence which is not monotone and the corresponding series is convergent.) In particular - construct examples justifying that assumptions of theorems cannot be omited.
   - Use definition to find supremum/limit/derivative/sum of a series...
   - Prove a statement not presented during lectures. (See some Problems 3 in homeworks for examples of such a task.)

List of required definitions

Definitions marked by $*$ are "obligatory". That is, not knowing (or making a major error in) such a definition during the exam will result into a "fail" mark, regardless of the rest of the exam.

- $*$ upper and lower bound, supremum, infimum
- set bounded from below/above
- $*$ countable set
- field
- completeness axiom
- $*$ proper and improper limit of a sequence, convergent and divergent sequence
- monotone, non-increasing, decreasing, non-decreasing, increasing sequencene
- sequence bounded from above/below

- Cauchy sequence
- accumulation points of a sequence, limes inferior, limes superior
- $*$ series, sum of a series, convergence and divergence of a series
- absolute convergence
- rearrangement of a series
- exponential function
- sinus, cosinus
- neighborhood, reduced neighborhood, left and right (reduced) neighborhood of a point
- $*$ limit of a function at a point
- $*$ continuity of a function at a point and on an interval
- one sided limit
- local and global extremes of a function
- inverse function
- $*$ derivative of a function at a point
- higher order derivatives
- convexity and concavity
- inflection point
- Taylor polynomial and series

List of required theorems (or lemmas)

- Bernoulli inequality.
- Irrationality of $\sqrt{2}$.
- Cantor theorem - set of reals is uncountable.
- Existence of suprema a infima of a bounded set in $\mathbb{R}$.
- Uniqueness of a limit.
- Limit of a monotone sequence.
- Limit of a subsequence.
- Arithmetics of limits (including improper limits). (∂)
- Limit and ordering.
- Sandwich theorem.
- Monotone subsequence.
- Bolzano-Weierstrass.
- Cauchy property.
- About limsup a liminf.
- Divergence of harmonic series.
- Necessary condition for convergence of a series.
- Cauchy condition for series.
- Linear combination of series,
- Comparison criterion.
- D'Alembert criterion/ratio test.
- Cauchyovo criterion/root test.
- Absolute convergence implies convergence.
- Leibniz criterion.
- Riemann rearrangement theorem.
- Rearrangement of absolutely convergent series.
- Properties of the function e^x.

- Uniqueness of a limit of a function.
- Heine theorem.
- Aritmetics of limits of functions.
- Limit of monotone function.
- Limit of a function and ordering.
- Limit of a composed function.
- Darboux theorem.
- Continuous image of interval.
- Extremes of continuous function.
- Continuity of the inverse function.
- Arithmetics of derivatives.
- Derivative of a composed function.
- Derivative of an inverse function.
- Necessary condition for local extreme.
- Derivative and monotonicity.
- Limit and derivative.
- Derivative and continuity.
- Mean value theorems.
- L'Hospital rule.
- Characterisation of Taylor polynomial.
- Remainder of Taylor polynomial.
- Convexity and second derivative.
- Necessary condition for inflection.
- Sufficient condition for inflection.

Please let me know if you spot any error in the lists or have any questions.