Mathematical analysis I — Tutorial 2

http://kam.mff.cuni.cz/~tereza/teaching.html

Problem 1: Prove that for any sets A, B, C we have that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. The proof should be in two parts: first show that if $x \in A \cup (B \cap C)$ then $x \in (A \cup B) \cap (A \cup C)$, then show the reverse. You should only use the definitions of $\cup$ and $\cap$.

Problem 2: In real numbers, determine suprema and infima of the following sets (if they exist). Are these minima and maxima of the sets?:

a) $M = \left\{-\frac{1}{n}, n \in \mathbb{N}\right\}$

b) $M = \left\{\frac{p}{p+q}, p, q \in \mathbb{N}\right\}$

c) $M = \left\{\frac{n+(-1)^n}{n}, n \in \mathbb{N}\right\}$

d) $M = \left\{n^{(-1)^n}, n \in \mathbb{N}\right\}$

e) $M = \{2^{-n} + 3^{-n} : n \in \mathbb{N}\}$

f) $M = \{2^{-n} + 3^{-n} : n \in \mathbb{Z}\}$

g) $M = \{5^{(-1)^j 3^k} : j, k \in \mathbb{Z}\}$

Problem 3: On the sets of integers ordered by inclusion, determine supremum and infimum of the following set. $M = \{\{1,3,5,7,9\}, \{2,3,5,9\}, \{1,5,7,9,10\}, \{4,5,6,9,11\}\}$.

Problem 4: For bounded nonempty sets of reals A a B, describe suprema and infima of the following sets as exactly as possible, using suprema and infima of A and B.

a) $A \cup B$

b) $A \cap B$, assuming it is nonempty.

c) $A \setminus B$, assuming it is nonempty.

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