Mathematical analysis I — Limits of functions

Limits to remember:

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{e^x-1}{x}=1 \qquad \lim_{x\to 0}\frac{\ln(1+x)}{x}=1$$

Problem 1: Find limits:

a) $\lim_{x\to\pi}\frac{\sin\frac{x}{2}}{x}$

b) $\lim_{x\to 1}\frac{x^2+2x-3}{x^2-1}$

c) $\lim_{x\to 1}\frac{x^3-1}{(x-1)^2}$

d) $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

e) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

f) $\lim_{x\to 0}\frac{1}{x^2}$

g) $\lim_{x\to\infty}\frac{\ln(1+2^x)}{x}$

h) $\lim_{x\to 0}(2e^x-1)^{\frac{x^2+1}{x}}$

i) $\lim_{x\to 0}\frac{\ln(1+\sin x)}{x}$

Mathematical analysis I — Homework 11

Due: 15:40, 19.12.

Write your solution of each problem on a separate sheet of paper. One part will be marked for credit.

Problem 1: Find the following limits (don't use L'Hospital's rule):

(a) $\lim_{x\to 1}\frac{x-1}{\sqrt{x+3}-2}$

(b) $\lim_{x\to 0}\frac{\tan(2x)}{x}$

Problem 2: Using definition of a limit, show that the function $f(x)=\sqrt{x}$ is continuous at a point a for every $a\in[0,\infty)$. That is, show that $\lim_{x\to a}\sqrt{x}=\sqrt{a}$ for every $a\in[0,\infty)$. Hint: Use that every $a,x\geq 0$, unless $x=a=0$, satisfy $\sqrt{x}-\sqrt{a}=(\sqrt{x}-\sqrt{a})\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}$.

Problem 3: Let $p(x)$ be a polynomial. Determine $\lim_{x\to-\infty}(p(x+1)-p(x))$.