Mathematical analysis I — Plotting graphs, l'Hospital rule

Problem 1: Plot a graph of a function. Check list:

- domain of the function
- is it periodic/odd/even
- intersections with axis (if they are reasonably easy to find)
- points where function is continuous and discontinuous
- limits at points of discontinuity (possibly one sided) and at $\pm\infty$
- derivative: extremes and intervals where function is increasing/decreasing
- range of the function (using knowledge of extremes and discontinuities)
- second derivative: inflection points and convexity/concavity

a) $\frac{|2x-1|}{(x-1)^2}$

b) $\arcsin\left(\frac{2x}{x^2+1}\right)$

c) $\sin(x) + \cos(x)$

Problem 2: Find limits of the functions using l'Hospital rule.

a) $\lim_{x\to 0} \frac{\ln\cos x}{x}$

b) $\lim_{x\to\infty} x(\pi/2 - \arctan x)$

c) $\lim_{x\to 0} \frac{e^{x^2}-1}{\cos x - 1}$

d) $\lim_{x\to\infty} \frac{e^{\sqrt{x}}}{x}$

e) $\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{\sin^2 x}$

f) $\lim_{x\to\infty}(1 - \frac{2}{x})^x$