Basic limits

These limits can be used without further justification.

Let $\alpha, r \in \mathbb{R}$, then

$$\lim_{n\to\infty} n^\alpha = \begin{cases} +\infty & \text{for} \quad \alpha > 0 \\ 1 & \text{for} \quad \alpha = 0 \\ 0 & \text{for} \quad \alpha < 0 \end{cases} \tag{1}$$

$$\lim_{n\to\infty} \alpha^n = \begin{cases} +\infty & \text{for} \quad \alpha > 1 \\ 1 & \text{for} \quad \alpha = 1 \\ 0 & \text{for} \quad -1 < \alpha < 1 \\ \text{does not exist} & \text{for} \quad \alpha \leq -1. \end{cases} \tag{2}$$

$$\lim_{n\to\infty} \sqrt[n]{\alpha} = 1 \qquad \text{for } \alpha > 0 \tag{3}$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1 \tag{4}$$

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e \tag{5}$$

$$\lim_{n\to\infty} \frac{n^r}{\alpha^n} = 0 \qquad \text{for } \alpha > 1, r > 0 \tag{6}$$

$$\lim_{n\to\infty} \frac{\ln n}{n^\alpha} = 0 \qquad \text{for } \alpha > 0 \tag{7}$$