Mathematical analysis II — Tutorial 8

http://kam.mff.cuni.cz/~tereza/teaching.html

Problem 1: A function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is defined as

$$f(x,y) = \sqrt{|x||y|}.$$

Check that f is continuous at $(0,0)$, and has partial derivatives and argue that$f$ is not differentiable at $(0,0)$

Problem 2: Verify equality $\frac{\partial^2 f(x,y)}{\partial x \partial y} = \frac{\partial^2 f(x,y)}{\partial y \partial x}$ for $f(x,y) = x^3 + 4xy - y^2$

Problem 3: Write down the Jacobian matrices of the following composite functions:

a) $f(u,v) = (u^2v^2, 1/(uv)), g(x,y) = \ln x + \ln y, h = g \circ f$

b) $g(t) = (\sin t, \cos t), f(x,y) = x + y, h = f \circ g$

c) $f(u,v) = (\sin uv, \cos uv), g(x,y) = x^2 + y^2, h = g \circ f.$

Problem 4: Assume that f is differentiable at $(1,1)$ and $g(t,u) = f(f(u,t), f(t,u))$. Find $\partial_1 g(1,1)$ if $f(1,1) = \partial_1 f(1,1) = 1$, $\partial_2 f(1,1) = 2$.

Mathematical analysis II — Homework 9

Due: 9:00, 24.4.2019

Write your solution of each problem on a separate sheet of paper of format A4, without torn edges. One part will be marked for credit.

Problem 1: Find all second partial derivatives of a function f defined as $f(0,0) = 0$ and $f(x,y) = xy\frac{x^2-y^2}{x^2+y^2}$ for $(x,y) \neq (0,0)$.

Problem 2: Let f be a function of two variables in polar coordinates r and θ. Express partial derivatives with respect to cartesian coordinates $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ using $\frac{\partial f}{\partial r}$, $\frac{\partial f}{\partial \theta}$, r and θ. Relation between polar and cartesian coordinates is:

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$r = \sqrt{x^2+y^2} \qquad \theta = \arctan\left(\frac{y}{x}\right).$$

Problem 3: Let $f(x,y) = \sqrt{1 - x^2 - y^2}$. Find the equation of the contour going through the point $(1/2, 1/2)$, and write down the tangent line of thecontour at the point $(1/2, 1/2)$. Calculate the gradient of the function at $(1/2, 1/2)$. What is the angle between the tangent line and the contour?