Mathematical analysis II — Tutorial 8

Problem 1: Find total differential of the following functions:

a) $(1+x)^k(1+y)^\ell$, where $k, \ell \in \mathbb{N}$ are parameters.

b) $\ln(1+x)\ln(1+y)$

c) $(1+x)^{1+y}$

Problem 2: Find directional derivative of $f(x,y) = \arctan xy$ at a point $(1,1)$ in the direction $(-1,-1)$.

Problem 3: In what direction is the directional derivative of $f(x,y) = (x-y)^2$ maximal or minimal at the point $(-4,2)$? (Note: "in which direction" asks for comparing directional derivatives of vectors with the same length(norm)). If you put a ball on the graph of f at the point $(-4,2)$, in which direction would it start rolling?

Problem 4: A function $f \colon \mathbb{R}^2 \to \mathbb{R}$ is defined as

$$f(x,y) = \sqrt{|x||y|}.$$

Check that f is continuous at $(0,0)$, and has partial derivatives and argue that f is not differentiable at $(0,0)$

Problem 5:

Find the tangent plane to the graph of the function $f(x,y) = x^y$ at the point $(2,3)$.

Mathematical analysis II — Homework 8

Due: 9:00, 17.4.2019

Write your solution of each problem on a separate sheet of paper of format A4, without torn edges. One part will be marked for credit.

Problem 1: Continuously extend the function $f(x,y) = \frac{x^3 y}{x^2 + y^2}$ to $\mathbb{R}^2$ and find total differential at all points of $\mathbb{R}^2$.

Problem 2: Find the tangent plane to the graph of the function $f(x,y) = x^2 + 2xy + 2y$ at the point $(1,2)$. At which points has the graph of f horizontal tangent plane?

Problem 3: Which statement implies the other? Justify validity or invalidity of both implications.

P: $f(x,y)$ is continuous at $(0,0)$.

Q: The partial deriva-tives of $f(x,y)$ exist at $(0,0)$.