Mathematical analysis II — Tutorial 5

Problem 1: Find Riemann integral using the definition:

a) $\int_{-\pi}^{\pi} 1 \, dx$

b) $\int_{-1}^{1} \operatorname{sgn}(x) \, dx$

c) $\int_{0}^{3} \lfloor x \rfloor \, dx$

Problem 2: Find definite integral:

a) $\int_{0}^{1} x^{\alpha} \, dx$

b) $\int_{0}^{\pi} \sin x \, dx$

c) $\int_{0}^{\infty} \sin x \, dx$

d) $\int_{-\sqrt{3}/2}^{0} \frac{1}{\sqrt{1-x^2}} \, dx$

e) $\int_{0}^{8} \sqrt{1+x} \, dx$

f) $\int_{0}^{1} \frac{1}{3-x} \, dx$

g) $\int_{1/e}^{e} |\ln x| \, dx$

h) $\int_{0}^{\infty} e^{-x} \, dx$

i) $\int_{0}^{1} \frac{1}{\sqrt{x}} \, dx$

Problem 3: Find definite integral using the per partes method:

a) $\int_{0}^{\pi/2} \sin^{n} x \, dx$

b) $\int_{0}^{\infty} x^{n} e^{-x} \, dx$

c) $\int_{0}^{\pi} \frac{\sin 2x}{\sin x} \, dx$

Mathematical analysis II — Homework 5

Due: 9:00, 27.3.2019

Write your solution of each problem on a separate sheet of paper of format A4, without torn edges. One part will be marked for credit.

Problem 1: Calculate definite integrals (if they converge):

$$\int_{2}^{5} x^2 \sqrt{x^3 + 1} \, dx \qquad\qquad \int_{2}^{\infty} \frac{1}{x^3} \, dx \qquad\qquad \int_{0}^{1} \ln x \, dx$$

Problem 2: Show that $\int_{ca}^{cb} f(x) \, dx = c \int_{a}^{b} f(cx) \, dx$. From this, derive that $\int_{1}^{a} \frac{1}{x} \, dx + \int_{1}^{b} \frac{1}{x} \, dx = \int_{1}^{ab} \frac{1}{x} \, dx$.

Problem 3: Decide and justify whether functions $\ln x$ and x^2 are uniformly continuous on interval $(0, 1)$.