

Mathematical analysis II — Tutorial 4

<http://kam.mff.cuni.cz/~tereza/teaching.html>

Standard substitutions:

Rational functions in $\sin x$ and $\cos x$ $R(\sin x, \cos x)$:

- $R(u, v) = R(-u, -v)$ substitute $y = \operatorname{tg} x$ or $y = \operatorname{cotg} x$
- $R(-u, v) = -R(u, v)$ substitute $y = \cos x$
- $R(u, -v) = -R(u, v)$ substitute $y = \sin x$
- Weierstrass substitution $y = \operatorname{tg}(x/2)$ (universal)

Trigonometric substitutions:

- $f(a^2 - x^2)$ substitute $x = a \sin y$
- $f(a^2 + x^2)$ substitute $x = a \operatorname{tg} y$
- $f(x^2 - a^2)$ substitute $x = a / \cos y$

Square root substitutions:

- $R(x, \sqrt{\frac{ax+b}{cx+d}})$ substitute $y = \sqrt{\frac{ax+b}{cx+d}}$
- $R(x, \sqrt{ax^2 + bx + c})$ substitute $\sqrt{ax^2 + bx + c} = \sqrt{ax} + y$ (Euler substitution)

Problem 1: Find primitive function (use substitution to reduce the problem of finding a primitive function of a rational function):

a) $\int \frac{x-1}{x(\sqrt{x} + \sqrt[3]{x})} dx$

d) $\int \frac{dx}{\sqrt{x^2+4}}$

b) $\int \frac{dx}{\sqrt{x^2+x+1}}$

e) $\int \frac{dx}{1+\sin x}$

c) $\int \sqrt{a^2 - x^2} dx$

f) $\int \frac{dx}{1-\sin^2 x}$

Mathematical analysis II — Homework 4

Due: 9:00, 20.3.2019

Write your solution of each problem on a separate sheet of paper of format A4, without torn edges. One part will be marked for credit.

Problem 1: Find primitive functions using substitutions mentioned above:

$$\int \frac{1}{1 + \sqrt{x+1}} dx \quad \int \sin^7 x dx \quad \int \frac{1}{(1+x^2)^2} dx$$

Problem 2: Find a primitive function of $x^3\sqrt{1-x^2}$ using trigonometric substitution.

Problem 3: Calculate $\int \frac{1}{\cos x} dx$ in two ways:

- using Weierstrass substitution
- using partial fractions and the fact that $\frac{1}{\cos x} = \frac{\cos x}{\cos^2 x} = \frac{\cos x}{1-\sin^2 x}$