Mathematical analysis II — Tutorial 3

http://kam.mff.cuni.cz/~tereza/teaching.html

Problem 1: Complete the following table (including intervals where the primitive function is defined). For the last function, find a recurrence using integration by parts.

$f(x)$	$F(x)$ (constant omitted)
$\frac{1}{x-\alpha}$	$\ln\lvert x-\alpha\rvert$
$\frac{1}{(x-\alpha)^k}$; $k>1$	$-\frac{1}{(k-1)x^{k-1}}$
$\frac{2x+p}{x^2+px+q}$	$\ln\lvert x^2+px+q\rvert$
$\frac{1}{x^2+px+q}$; $q>\frac{p^2}{4}$	$\operatorname{arctg}\left(\frac{x+p/2}{\sqrt{q-p^2/4}}\right)/\sqrt{q-p^2/4}$
$\frac{2x+p}{(x^2+px+q)^k}$; $k>1$	$-\frac{1}{(k-1)(x^2+px+q)^{k-1}}$
$\frac{1}{(x^2+1)^{k+1}}$; $k\geq 1$	$\left(\frac{x}{(1+x^2)^n}+(2n-1)\int\frac{1}{(x^2+1)^k}\,dx\right)/(2n)$

Problem 2: Find all primitive functions of the function:

$$f(x)=x+1+\frac{3}{2x-1}+\frac{1}{(2x-1)^3}+\frac{x+2}{x^2+2x+3}+\frac{1}{(x^2+2x+3)^2}$$

Problem 3: Decompose the following functions into partial fractions.

a) $\frac{1}{x(x-1)}$

b) $\frac{4}{(x+2)(2x+1)}$

c) $\frac{x+1}{x^2+x-6}$

d) $\frac{x^3}{(x-2)^2}$

e) $\frac{2x+5}{x^3-6x^2-6x-7}$

Problem 4: Find primitive functions (on maximal intervals):

a) $\int\frac{x^7-5}{x^2-1}\,dx$

b) $\int\frac{x^4}{x^4+5x^2+4}\,dx$

c) $\int\frac{x}{x^3-3x+2}\,dx$

Mathematical analysis II — Homework 3

Due: 9:00, 13.3.2019

Write your solution of each problem on a separate sheet of paper of format A4, without torn edges. One part will be marked for credit.

Problem 1: Find a primitive functions and determine on which intervals are they defined:

$$\int\frac{e^x}{e^x+1}\,dx \qquad \int\frac{1}{x(\ln^2 x-5\ln x+6)}\,dx \qquad \int\frac{\sin x}{\cos^2 x-1}\,dx$$

Problem 2: Find a primitive function of $\frac{-2x+4}{x^4-2x^3+2x^2-2x+1}$ and determine on which intervals is it defined.

Problem 3: Let $P(x)$ be a polynomial and $Q(x)$ a polynomial in the form $\prod_{i=1}^n(x-\alpha_i)^{k_i}$, where $\alpha_i\in\mathbb{R}$ and $k_i\in\mathbb{N}$ for every $i=1,\dots,n$. We define $Q_1(x)=\prod_{i=1}^n(x-\alpha_i)^{k_i-1}$ and $Q_2(x)=\prod_{i=1}^n(x-\alpha_i)$. Show that there are polynomials $P_1(x)$ and $P_2(x)$, such that

$$\int\frac{P(x)}{Q(x)}\,dx=\frac{P_1(x)}{Q_1(x)}+\int\frac{P_2(x)}{Q_2(x)}\,dx.$$