$$\frac{d\tan(x)}{dx} = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos^2 x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(x^k)' = kx^{k-1} \qquad k \in \mathbb{R}$$

$$\frac{d\sin(\overbrace{x^2}^{=:y})}{dx} = \frac{d\sin(y)}{dy}\,\frac{d\overset{x^2}{y}}{dx} = \cos(y)\,2x = \cos(x^2)\,2x$$

$$\frac{d}{dx}\sqrt{\frac{x+1-1+1}{x-1}}' = \frac{d}{dx}\left(1+\frac{2}{x-1}\right)^{\frac{1}{2}}$$

$$\frac{d}{dx}\sqrt{\frac{x+1-1+1}{x-1}} = \frac{d}{dx}\left(1+\frac{2}{x-1}\right)^{\frac{1}{2}} = \frac{d\sqrt{z}}{dz}\,\frac{dz}{dy}\,\frac{dy}{dx} =$$

$$z = 1 + y \qquad y = \frac{2}{x-1}$$

$$= \frac{1}{2} z^{-\frac{1}{2}} (0+1) \frac{d}{dx} \frac{2}{x-1} = \frac{1}{2\sqrt{z}}\, 2 \frac{d}{dx} (x-1)^{-1} = \frac{1}{\sqrt{z}} (-1)(x-1)^{-2} \overbrace{\frac{d(x-1)}{dx}}^{=1}$$

$$= \underline{\frac{-1}{\sqrt{1+\frac{2}{x-1}}}\;\frac{1}{(x-1)^2}}$$

$$\lim_{x \to 0} \frac{1-\cos(x)}{x^2} = \lim_{x \to 0} \frac{0-(-\sin(x))}{2x} = \lim_{x \to 0} \frac{1}{2} \frac{\sin x}{x} = \frac{1}{2} \lim_{x \to 0} \frac{\cos(x)}{1}$$

$$\lim_{x \to 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}$$

$$\frac{d}{dx}\frac{(\ln x)^x}{x^{\ln x}} = \frac{d}{dx}\frac{(\ln x)^x}{e^{\ln x \ln x}} = \frac{d}{dx}\underbrace{\frac{(\ln x)^x}{e^{\ln^2 x}}} =$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$e^y \cdot e^z = e^{y+z}$$

$$e^{x \ln\ln x} \cdot e^{-\ln^2 x} = e^{x\ln\ln x - \ln^2 x}$$

$$= \frac{de^y}{dy}\frac{dy}{dx} =$$

$$= e^y \frac{d(x\ln\ln x - \ln^2 x)}{dx} = e^y\left(\frac{d\, x\ln\ln x}{dx} - \frac{d\ln^2 x}{dx}\right) =$$

$$= e^y\left(\frac{dx}{dx}\ln\ln x + x\frac{d\ln \overbrace{\ln x}^{\tilde{y}}}{dx} - 2\ln x\frac{d\ln x}{dx}\right) =$$

$$= e^y\left(\ln\ln x + x\underbrace{\frac{d\ln\tilde{y}}{d\tilde{y}}\frac{d\tilde{y}}{dx}} - 2\ln x\frac{1}{x}\right) =$$

$$\frac{d\ln\tilde{y}}{d\tilde{y}}\frac{d\ln x}{dx} = \frac{1}{\tilde{y}}\frac{1}{x} = \frac{1}{x(\ln x)}$$

$$= e^{x\ln\ln x - \ln^2 x}\left(\ln\ln x + \frac{1}{\ln x} - \frac{2\ln x}{x}\right)$$

$$\frac{d}{dx}\left(x^2 \ln x \arctan x\right) = (x^2)' \ln x \operatorname{arctg} x + x^2 (\ln x \arctan x)' =$$

$$= (x^2)' \ln x \arctan x + x^2 (\ln x)' \arctan x + x^2 \ln x (\arctan x)' =$$

$$= 2x \ln x \arctan x + x^2 \frac{1}{x} \arctan x + x^2 \ln x \frac{1}{1+x^2}$$

$$\lim_{x\to\frac{\pi}{4}} (\tan x)^{\tan 2x} = \lim_{x\to\frac{\pi}{4}} e^{\tan(2x)\ln(\tan x)} =$$

$$= \lim_{x\to\frac{\pi}{4}} e^{\frac{\ln(\tan x)}{\frac{1}{\tan(2x)}}} \overset{l'H}{=} \lim_{x\to\frac{\pi}{4}} e^{\frac{\frac{1}{\tan x}\,\frac{1}{\cos^2(x)}}{-\frac{1}{\tan^2(2x)}\,\frac{1}{\cos^2(2x)}\,2}} =$$

$$= \lim_{x\to\frac{\pi}{4}} e^{\frac{\frac{1}{\cos^2 x\tan x}}{-\frac{\cos^2(2x)}{\sin^2(2x)}\,\frac{1}{\cos^2(2x)}\,2}} = \lim_{x\to\frac{\pi}{4}} e^{\frac{\frac{1}{\tan x\cos^2 x}}{-\frac{2}{\sin^2(2x)}}} =$$

$$= e^{\frac{\frac{1}{1\cdot\frac{1}{2}}}{-\frac{2}{1}}} = e^{-1}$$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \frac{df}{dx}$$