Mějme nyní reálnou posloupnost $(a_n) \subset \mathbb{R}$ a $a \in \mathbb{R}$. Číslo a nazveme (vlastní) limitou posloupnosti (a_n), pokud

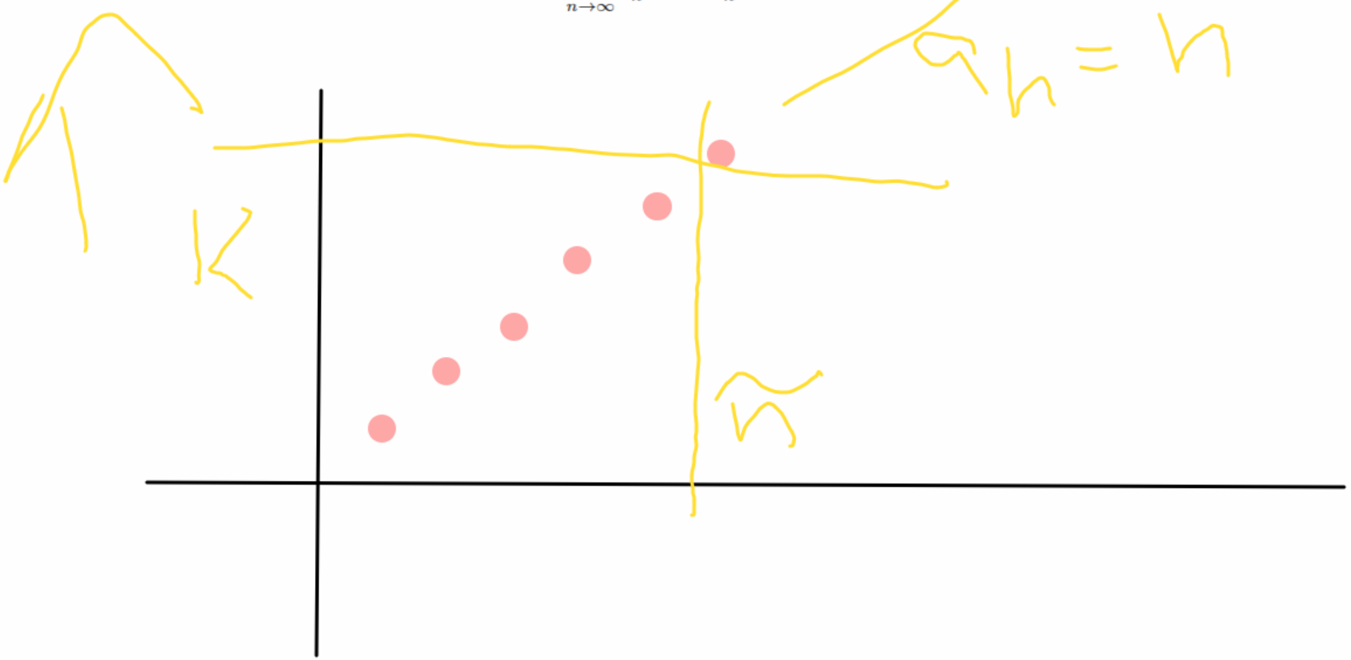
$$\forall \epsilon > 0\ \exists \tilde{n}\ \forall n > \tilde{n} : |a - a_n| < \epsilon.$$

Řekneme, že posloupnost má nevlastní limitu $\pm\infty$, pokud

$$\forall K \in \mathbb{R}\ \exists \tilde{n}\ \forall n > \tilde{n} : \pm a_n > K.$$

K označení používáme

$$\lim_{n\to\infty} a_n = \lim a_n = a.$$

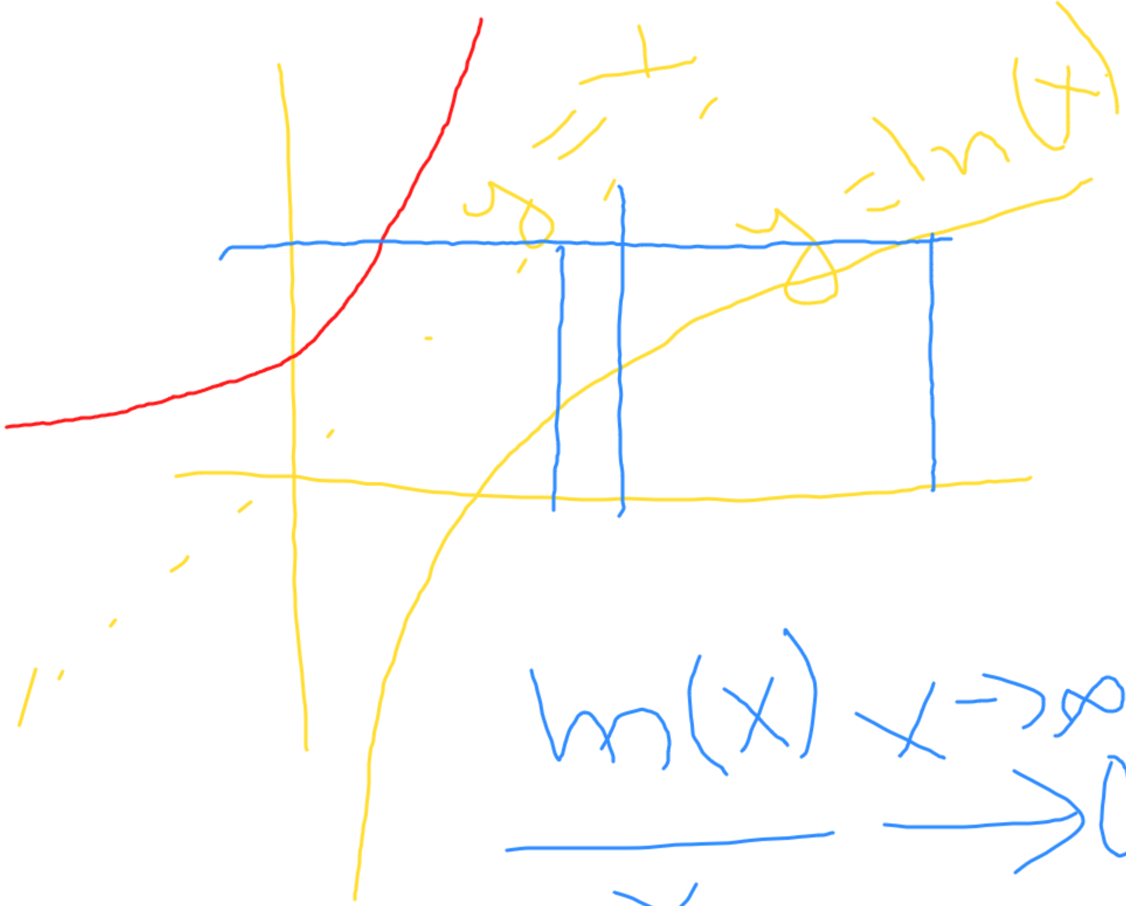


$$\ln(n) \rightarrow \infty$$

$$a_n > K$$

$$e^{\ln(n)} > e^K$$

$$\tilde{n} = \lceil e^K \rceil + 1$$



$$\frac{\ln(x)}{x} \xrightarrow{x \to \infty} 0$$

$$n! \geq \left(\frac{n}{2}\right)^{\frac{n}{2}}$$

$$n! = \sqrt{2\pi n}\; n^n \left(1+\frac{1}{n}\right)$$

$$n! \to \infty$$

$$a_n > K$$

$$n! \geq n > K$$

$$\tilde{n} = \lceil K \rceil + 1$$

$$\frac{1}{1+n^2} \longrightarrow 0$$

$$|a - a_n| < \varepsilon$$

$$\left|0 - \frac{1}{1+n^2}\right| < \varepsilon$$

$$\frac{1}{1+n^2} < \varepsilon$$

$$1 < \varepsilon(1+n^2)$$

$$\frac{1}{1+n^2} \leq \frac{1}{1+n}$$

$$\sqrt{\frac{1-\varepsilon}{\varepsilon}} < n$$

$$\sqrt{\frac{1}{\varepsilon} - 1} < n$$

$$\tilde{n} = \left\lceil \sqrt{\frac{1}{\varepsilon} - 1} \right\rceil + 1$$

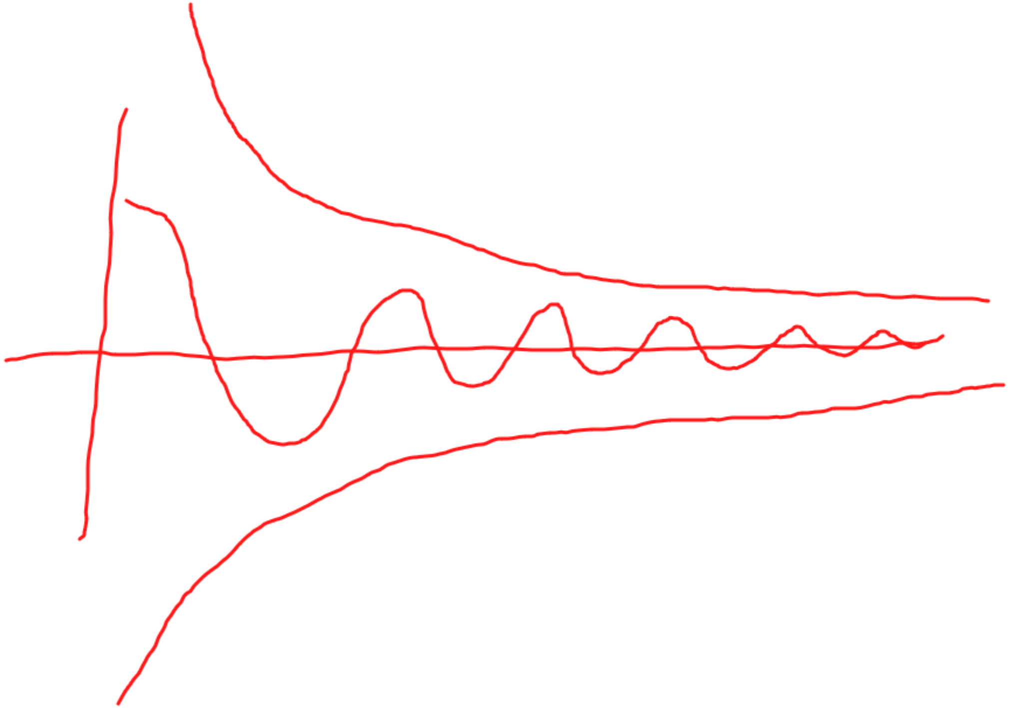
$$\lim_{n\to\infty} \sqrt{n+1} - \sqrt{n} = \lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right) \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}+1-\cancel{n}}{\sqrt{n+1} + \sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0$$

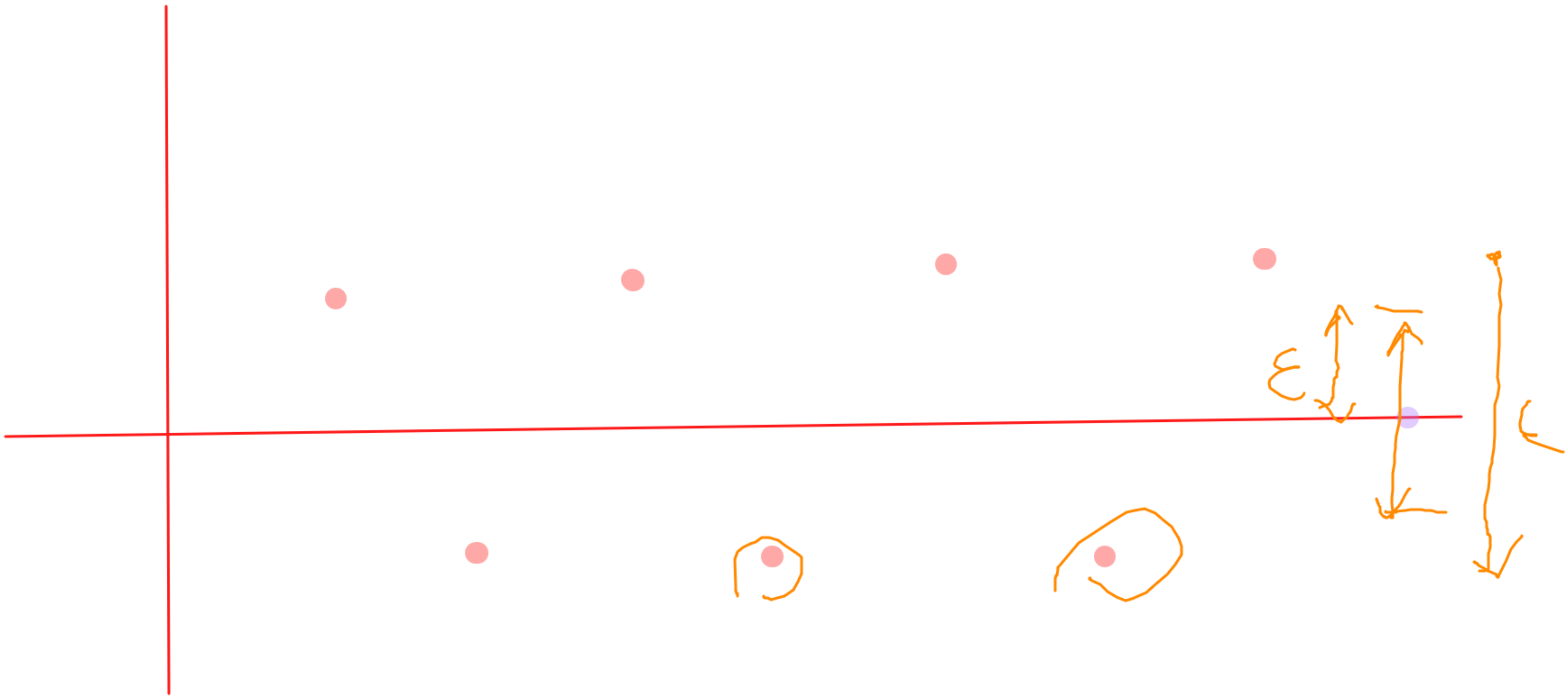
$$(a-b)(a+b) = a^2 - b^2$$

$$\lim_{n\to\infty} \frac{\sin(n)}{n} \longrightarrow 0$$



$$-\frac{1}{n} \le \frac{\sin(n)}{n} \le \frac{1}{n}$$

$$|a - a_n| = \frac{|\sin(n)|}{n} < \frac{1}{n} < \varepsilon$$



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