2c) $$\int |x|\,dx = \begin{cases} \int x\,dx, & x \geq 0 \\ -\int x\,dx, & x < 0 \end{cases}$$

$$= \begin{cases} \frac{x^2}{2} + C_1, & x \geq 0 \\ -\frac{x^2}{2} + C_2, & x \leq 0 \end{cases}$$

$$= \frac{x|x|}{2} + C$$

3a) $$\int x \sin(x)\,dx \overset{p.p.}{=} \left| \begin{matrix} F = \frac{x^2}{2} & G = \sin(x) \\ f = x & g = \cos(x) \end{matrix} \right| =$$

$$= \frac{x^2}{2}\sin(x) - \int \frac{x^2}{2}\cos(x)\,dx$$

$$\overset{p.p.}{=} \left| \begin{matrix} F = x & G = -\cos(x) \\ f = 1 & g = \sin(x) \end{matrix} \right| =$$

$$= x(-\cos(x)) - \int 1(-\cos(x))\,dx =$$

$$= -x\cos(x) + \int \cos(x)\,dx = -x\cos(x) + \sin(x) + c$$

4a) $\int \frac{e^{2x}}{1+e^{2x}}\,dx = \left\{ \begin{array}{l} 1+e^{2x} = y \\ \underbrace{(1+e^{2x})'}_{2e^{2x}}dx = dy \end{array} \right\} =$

$= \int \frac{1}{y} \frac{dy}{2} = \frac{1}{2}\int \frac{1}{y}\,dy = \frac{1}{2}\ln(y) + c =$

$= \frac{1}{2}\ln|1+e^{2x}| + c$

$\frac{3x+8}{x(x+2)} = \frac{4}{x} - \frac{1}{x+2}$

5a) $\int \frac{1}{1-x^2}\,dx =$

$= \frac{1}{1-x}\underbrace{\left(A + \overbrace{B\frac{(1-x)}{1+x}}\right)}_{x=1}$

$\frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)} = \overbrace{\frac{A}{1-x} + \frac{B}{1+x}} = \frac{A(1+x)+B(1-x)}{(1-x)(1+x)} =$

$= \frac{A+B+x(A-B)}{(1-x)(1+x)}$ $\left| \begin{array}{l} 1 = A+B \quad \Rightarrow A = \frac{1}{2} = B \\ 0 = A-B \Rightarrow A=B \end{array} \right.$

$\int \frac{1}{1-x^2}dx = \int \frac{1}{2}\frac{1}{1-x} + \frac{1}{2}\frac{1}{1+x}\,dx =$

$= \frac{1}{2}\int\frac{1}{1-x}\,dx + \frac{1}{2}\int\frac{1}{1+x}\,dx = \left\{ \begin{array}{l} 1-x = y \\ -dx = dy \end{array} \right\}$

$= \frac{1}{2}\int\frac{-dy}{y} +$ ——— " ——— $=$

$= -\frac{1}{2}\ln|y| +$ ——— " ——— $=$

$= -\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| + c$

$$f(x) = y$$
$$\frac{df(x)}{dx}dx = dy$$

6g) $$\int \frac{1}{x\ln(x)}\,dx = \left\{ \begin{array}{l} \ln(x) = y \\ \frac{1}{x}\,dx = dy \end{array} \right\} =$$

(dy)

$$= \int \frac{1}{y}\,dy = \ln|y| + c = \ln|\ln(x)| + c$$

6c) $$\int \operatorname{arctg}(x)\,dx \overset{pp.}{=} \left| \begin{array}{ll} F = x & G = \operatorname{arctg}(x) \\ f = 1 & g = \frac{1}{1+x^2} \end{array} \right| =$$

$$= x\operatorname{arctg}(x) - \int \frac{x}{1+x^2}\,dx = \left\{ \begin{array}{l} 1+x^2 = y \\ 2x\,dx = dy \end{array} \right\} =$$

$$= x\operatorname{arctg}(x) - \frac{1}{2}\int \frac{dy}{y} =$$

$$= x\operatorname{arctg}(x) - \frac{1}{2}\ln|1+x^2| + C$$

6h) $$\int \sin^5(x)\,\underbrace{\cos(x)\,dx} = \left\{ \begin{array}{l} \sin(x) = y \\ \cos(x)\,dx = dy \end{array} \right\} =$$

$$= \int y^5\,dy = \frac{y^6}{6} + C = \frac{\sin^6(x)}{6} + C$$

6k) $\int \frac{x}{1+x^4}\,dx = \left\{ \begin{array}{l} x^2 = y \\ 2x\,dx = dy \end{array} \right\} =$

$= \frac{1}{2} \int \frac{dy}{1+y^2} = \frac{1}{2} \operatorname{arctg}(y) + C =$

$= \frac{1}{2} \operatorname{arctg}(x^2) + C$