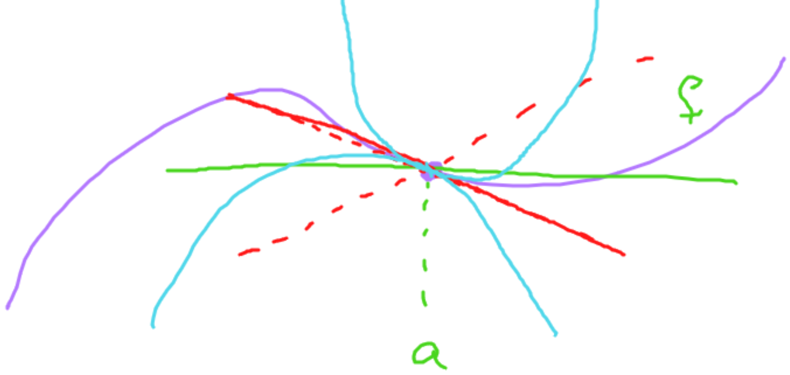
$$P_k = a_0 + a_1 x^1 + \cdots a_k x^k$$

$$P_k = \underset{\substack{\parallel \\ f(a)}}{a_0} + a_1(x-a)^1 + \cdots + a_k(x-a)^k$$



$$P_0 = f(a) \qquad f'(a)$$

$$P_1 = f(a) + a_1''(x-a)$$

$$f'(a) = P_1'(a) = (f(a) + a_1(x-a))' = a_1$$

$$P_2 = f(a) + f'(a)(x-a) + a_2(x-a)^2$$

$$P_2' = f'(a) + 2a_2(x-a)$$

$$P_2'' = 2a_2 = f''(a)$$

$$\Rightarrow a_2 = \frac{f''(a)}{2!}$$

$$P_3 = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + a_3(x-a)^3$$

$$P_3'(a) = f'(a) \qquad P_3''(a) = \frac{f''(a)}{2}$$

$$P_3''(x) = f''(a) + 3.2\, a_3(x-a)$$

$$f'''(a) = P_3'''(a) = 0 + 3.2.1\, a_3 = 3!\, a_3$$

$f = \ln(1+x^2)$

$P_0 = f(x_0) = \ln(1+x_0^2) = \ln(1) = 0$

$P_1 = 0 + \frac{f^{(1)}(x_0)}{1!}(x-x_0) =$

$f'(x) = \frac{2x}{1+x^2} \overset{x=0}{=} 0$

$P_2 = 0 + 0 + \frac{f''(x_0)}{2!}(x-x_0)^2$

$f''(x) = \frac{2(1+x^2) - 2x\,2x}{(1+x^2)^2} = \frac{2-2x^2}{(1+x^2)^2} \overset{x=0}{=} 2 > 0$

$$\lim_{x\to 0} \frac{a^x + a^{-x} - 2}{x^2} =$$

$$a^{-x} = e^{-\ln a\, x} = 1 - \ln a\, x + \ln^2 a \frac{x^2}{2!} + o(x^2)$$

$$a^x = e^{\overbrace{(\ln a)\, x}^{y}} = 1 + y + \frac{y^2}{2} + \frac{y^3}{3!} + \cdots$$

$$= 1 + \ln a\, x + \ln^2 a \frac{x^2}{2!} + o(x^2)$$

$$= \lim_{x\to 0} \frac{1 + \ln a\, x + \ln^2 a \frac{x^2}{2} + 1 - \ln a\, x + \ln^2 a \frac{x^2}{2} - 2 + o(x^2)}{x^2} =$$

$$= \lim_{x\to 0} \ln^2 a \frac{\frac{x^2}{2} + \frac{x^2}{2} + o(x^2)}{x^2} = \lim_{x\to 0} \ln^2 a \frac{x^2 + o(x^2)}{x^2} = \underline{\ln^2 a}$$

$\sqrt[5]{250}$ $\qquad\qquad$ $\sqrt[5]{x}$ $\qquad\qquad$ $\underline{\underline{243 = 3^5}}$

$$\sqrt[5]{243+7} = \sqrt[5]{3^5+7} = 3\sqrt{1+\frac{7}{3^5}}$$

$\sqrt[5]{x}$ $\qquad$ $x_0 = 243 = 3^5$

$$P = 3 + \frac{1}{5}(x)^{-\frac{4}{5}}\Big|_{x=243} \cdot (x-243) + o(x)$$

$$= 3 + \frac{1}{5\cdot 81}(x-243) + \dots$$

$$= 3 + \frac{7}{5\cdot 81}$$

$$\sin(\underbrace{\sin(x)}_{y})$$

$$\sin(y) = y - \frac{y^3}{3!} + \frac{y^5}{5!} + \dots$$

$$\frac{\sin x}{x} = 1$$

$$\sin(\sin(x)) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)\right)$$

$$-\frac{1}{3!}\left(x - \frac{x^3}{3!} + o(x^4)\right)^3 + \frac{1}{5!}\left(x - \frac{x^3}{3!} + o(x^4)\right)^5 =$$

$$= 0 \cdot x^0 + x^1(1) + x^2 \cdot 0 + x^3 \cdot \left(-\frac{1}{3!} - \frac{1}{3!}\right) + \dots$$

$$= x - \frac{2}{3!}x^3 + o(x^4)$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2} + \cdots$$

$$e^y = 1 + y + \frac{y^2}{2} + \cdots$$

$$\lim_{x\to\infty} x^4\left(\cos\left(\frac{1}{x}\right) - e^{-\frac{1}{2x^2}}\right)$$

$$e^{ix} = \cos(x) + i\sin(x)$$

$$\frac{1}{x} = y$$

$$\lim_{y\to 0^+} \frac{1}{y^4}\left(\cos(y) - e^{-\frac{y^2}{2}}\right) = \lim_{y\to 0^+} \frac{1}{y^4}\left(\frac{y^4}{4!} - \frac{y^4}{8} + o(y^4)\right) = \frac{1}{4!} - \frac{1}{8}$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} +$$

$$\cos(y) = 1 - \frac{y^2}{2!} + \frac{y^4}{4!} + o(y^4)$$

$$e^{-\frac{y^2}{2}} = 1 - \frac{y^2}{2} + \frac{1}{2}\left(-\frac{y^2}{2}\right)^2 + o(y^4)$$

$x \to ix$

$$= 1 + ix - \frac{x^2}{2} - i\frac{x^3}{3!} + \ldots$$