Algorithmic game theory – Tutorial 3*

1 Pure Nash equilibria

The *best response* of player i to the strategy profile s_{-i} is a mixed strategy s_i^* such that $u_i(s_i^*; s_{-i}) \geq u_i(s_i'; s_{-i})$ for each strategy $s_i' \in S_i$ of i. A *Nash equilibrium* in $G = (P, A, u)$ is a strategy profile $(s_1, \ldots, s_n)$ such that s_i is a best response of player i to s_{-i} for every $i \in P$.

In *pure Nash equilibrium*, all strategies are pure. We recall that pure Nash equilibria do not have to exist, for example, consider the Rock-Paper-Scissors game. A Nash equilibrium is *symmetric* if all players follow the same strategy.

Observation 1 (Best response condition)**.** *In a normal-form game $G = (P, A, u)$ of n players, for every player $i \in P$, a mixed strategy s_i is a best response to s_{-i} if and only if all pure strategies in the support of s_i are best responses to s_{-i}.*

Exercise 1. *We have two players who want to split 100 CZK among them. The first one chooses $x \in \{0, \ldots, 100\}$ and offers that he keeps $100 - x$ CZK, while the other one gets x. The second player can set a limit y of the smallest offer that he is willing to accept. If $x \geq y$, the offer is accepted and the players get payoffs $100 - x$ and x. If $x < y$, then the offer is rejected and both players get a payoff of 0.*

(a) *Play this game with the person next to you (you can play repeatedly and do not have to use real money). What x and y have you chosen?*

(b) *What are pure Nash equilibria of this game?*

Exercise 2. *Consider the following symmetric game of two players. Each player has the action set $\{2, 3, 4, 5, 6\}$. After choosing a number, both players get a payoff 0 if the chosen numbers are equal. If the numbers differ and one divides the other, then the player with the smaller number gets a payoff -2 and the other one gets 2. In any other case, the player with the larger number gets $+1$ and the other one gets -1.*

(a) *Play this game with a person next to you. Which (potentially mixed) strategies have you chosen?*

(b) *Find a pure Nash equilibrium of this game.*

Exercise 3. *We have $n \geq 2$ companies where each company chooses a real number $q_i \in [0, 50]$ indicating how much of a good will be produced. The revenue if the ith company then equals $q_i \cdot P$, where $P = 100 - (q_1 + \cdots + q_n)$.*

(a) *Play this game with a person next to you or in a larger group. You can play it repeatedly.*

(b) *Find a symmetric pure Nash equilibrium of this game.*

Exercise 4. *Consider the following game of $n \geq 2$ players. Every player selects, independently, a number from $\{1, \ldots, 1000\}$. The goal of each player is to have their number closest to the half of the average of all the selected numbers. All players who are closest to the half of the average split evenly the payoff of 1.*

(a) *Play this game with a person next to you or in a larger group. You can play it repeatedly.*

(b) *Find all pure Nash equilibria in this game.*

(c) *How do the pure Nash equilibria in this game change if each player who is closest to the half of average receives a payoff of 1?*

*Information about the course can be found at http://kam.mff.cuni.cz/~sychrovsky/