Spring School 2019

Tomáš Masařík, Veronika Slívová (eds.)

Preface

Spring school on Combinatorics has been a traditional meeting organized for over 35 years for faculty and students participating in the Combinatorial Seminar at Faculty of Mathematics and Physics of the Charles University. It is internationally known and regularly visited by students, postdocs and teachers from our cooperating institutions in the DIMATIA network. As it has been the case for several years, this Spring School is supported by Computer Science Institute (IÚUK) of Charles University, the Department of Applied Mathematics (KAM) and by some of our grants (SVV, Progres). This year we are glad we can also acknowledge generous support by the RSJ Foundation.

The Spring Schools are entirely organized and arranged by our students. The topics of talks are selected by supervisors from the Department of Applied Mathematics (KAM) and Computer Science Institute (IÚUK) of Charles University as well as from other participating institutions. In contrast, the talks themselves are almost exclusively given by students, both undergraduate and graduate. This leads to a unique atmosphere of the meeting which helps the students in further studies and their scientific orientation.

This year the Spring School is organized in Deštné v Orlických horách (in Eagle Mountains in northeastern Bohemia) with a great variety of possibilities for outdoor activities.

Tereza Klimošová, Ondřej Pangrác, Robert Šámal, Martin Tancer

KAM CSI IÚUK RSJ

Contents

Schedule of talks

	9:00 – 10:30	10:30 – 12:00	12:00
Saturday *room A*	A Polynomial Excluded-Minor Approximation of Treedepth Wojciech Nadara 9:00 – 10:30 30	A bound on the inducibility of cycles Jana Novotná 10:30 – 12:00 32	Lunch 12:00 – 13:00
Saturday *room B*		NP-hardness and fixed-parameter tractability of the minimum spanner problem Jan Soukup 10:30 – 12:00 44	Lunch 12:00 – 13:00
Sunday *room A*	Monge property for interval matrices Martin Černý 9:00 – 10:30 15	Infinitude of Primes Using Formal Languages Petr Chmel 10:30 – 12:00 18	Lunch 12:00 – 13:00
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Thursday	Topics about d-regular graphs, d fixed: Random regular graphs (chapter of a book Random graphs) Matas Šileikis 9:00 – 10:30 6	Topics about d-regular graphs, d fixed: Almost every d-regular graph is d-connected (with room to spare) Aneta Šťastná 10:30 – 12:00 7	Lunch 12:00 – 13:00
Friday	A vertex ordering characterization of simple-triangle graphs Karolina Okrasa 9:00 – 10:30 33	Communication Complexity Lower Bound of Disjointness Pavel Koblich Dvořák 10:30 – 12:00 16	Lunch 12:00 – 13:00

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Series Talks

Matas Šileikis

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Presented paper by J. Janson, T. Łuczak, A. Ruciński

Random regular graphs (chapter of a book *Random graphs*) *as part of series* Topics about d-regular graphs

(https://onlinelibrary.wiley.com/doi/book/10.1002/9781118032718)

Introduction

In the talk we will consider a r-regular graph chosen uniformly at random: how to generate it, how to prove typical properties of it and how many r-regular graphs on n vertices there is. For fixed r all this relies on understanding the distribution of short cycles in a certain random multigraph known as the *configuration model*.

Definitions

Let $1 \le r = r(n) < n$ such that $rn = 2m$ is even and denote $\mathscr{G}_{n,r}$ be a set of all r-regular graphs on vertex set $[n] = \{1, 2, \ldots, n\}$. A graph chosen uniformly at random from $\mathscr{G}_{n,r}$ is denoted by $G_{n,r}$ and called *a random r-regular graph*.

A *configuration* F is a partition of the set $W = [n] \times [r]$ into $m = nr/2$ pairs of vertices, called *edges* of F and Φ denotes the *set of configurations*.

Given a configuration $F \in \Phi$, let $\varphi(F)$ be the multigraph with vertex set V in which $(i, j) \in E$ if and only if F has a pair with one element in W_i and the other in W_j. We write $X_k(F)$ for the number of k-tuple of edges in F such that $\varphi(F)$ forms a k-cycle. So $X_1(F)$ is the number of edges that correspond to a loop in $\varphi(F)$ and $X_2(F)$ is the number of pairs of edges corresponding to a multiple edge in $\varphi(F)$. Then we have

$$\varphi^{-1}(\mathscr{G}_{n,r}) = \{F \in \Phi : X_1(F) = X_2(F) = 0\}.$$

Given a sequence of events E_n, we say that E_n *holds asymptotically almost surely (a.a.s.)* if the probability $P(E_n) \to 1$ for $n \to \infty$.

A random variable Z has Poisson distribution with parameter $\lambda \ge 0$ if $P(Z = k) = \frac{\lambda^k}{k! e^\lambda}$.

Main result

Theorem 1 (Theorem 9.5 in [1]) *For $k = 1, 2, \ldots,$ let X_k be the number of k-cycles in a randomly chosen configuration. Then X_k converges to Poisson distribution with parameter $\lambda_k = \frac{1}{2k}(r-1)^k$, jointly for all k.*

In the talk I will define/remind what it means to converge in distribution (jointly).

We will see that the following is a corollary of Theorem 1.

Proposition 2 *If $r \ge 2$ is fixed and Q^* holds a.a.s. for a random configuration, then Q holds a.a.s. for the random r-regular graph.*

Bibliography

[1] Janson, Svante, Tomasz Luczak, and Andrzej Rucinski. *Random graphs.* John Wiley & Sons, 2011.

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Presented paper by Bollobás, Béla

Almost every d-regular graph is d-connected (with room to spare)
as part of series Topics about d-regular graphs

(https://doi.org/10.1017/CBO9780511814068)

Introduction

When speaking about connectivity of r-regular graphs, the best we can hope for them is to be r-connected. The truth is even better: for almost every r-regular graph holds that if there exists a small set separating it, then at least one of the components is rather small. We will create a cut-set of vertices using neighbourhood of small connected subraph of G. For example, a triangle is present with probability greater than 0 for $n \to \infty$, and so it can be used as the small connected subgraph.

Definitions

Let $1 \le r = r(n) < n$ and $rn = 2m$ and denote $\mathscr{G}_{n,r}$ be the set of all r-regular graphs on n vertices. A graph chosen uniformly at random from $\mathscr{G}_{n,r}$ is denoted by $G_{n,r}$ and called *a random r-regular graph*. We define

$$\Gamma(A) = \{y \mid xy \in E(G) \text{ for some } x \in A\}$$

Given a sequence of (probability) events E_n, we say that E_n *holds asymptotically almost surely (a.a.s.)* if the probability $P(E_n) \to 1$ for $n \to \infty$.

Denote $\mathscr{C}(n,m)$ the set of *connected* graphs $G = (V, E)$ such that $|V| = n$ and $|E| = m$.

Main theorem

The following theorem implies, in particular, that a.a.s. $G_{n,r}$ is r-connected.

Theorem 1 (Theorem 7.32 from [1]) *For $r \ge 3$ and $a_0 \ge 3$, asymptotically almost surely $G_{n,r}$ is such that if $V = A \cup S \cup B$, $a = |A| \le |B|, s = |S|$ and there is no $A - B$ edge, then*

$$s \ge r, \qquad \text{if } a = 1, \tag{1}$$
$$s \ge 2r - 3, \qquad \text{if } a = 2, \tag{2}$$
$$s \ge (r-2)a, \qquad \text{if } 3 \le a \le a_0 \tag{3}$$
$$\text{and } s \ge (r-2)a_0, \qquad \text{if } a \ge a_0. \tag{4}$$

Some helpful propositions

Theorem 2 (Theorem 5.20 from [1]) *There is an absolute constant c such that for $1 \le k \le n$*

$$|\mathscr{C}(n, n+k)| \le \left(\frac{c}{k}\right)^{\frac{n}{2}} n^{n+\frac{(3k-1)}{2}}$$

Bibliography

[1] Béla Bollobás. *Random Graphs*. Cambridge University Press, Cambridge, United Kingdom, 2001.

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Introduction *as part of series* Quantum computing

Definitions

Definition 1 Unitary matrix: *$A \in \mathbb{C}^{n\times n}$ is unitary iff $A^{-1} = A^*$ (transpose and take complex conjugates). Equivalent definition if for each $v \in \mathbb{C}^n$ we have $\|Av\| = \|v\|$.*

Definition 2 Tensor product *let $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{m'\times n'}$ then $A \oplus B \in \mathbb{C}^{mm'\times nn'}$ is the matrix*

$$\begin{pmatrix} A_{1,1}B & A_{1,2}B & \dots & A_{1,n}B \\ A_{2,1}B & A_{2,2}B & \dots & A_{2,n}B \\ \dots & & & \\ A_{n,1}B & A_{n,2}B & \dots & A_{n,n}B \end{pmatrix}$$

Thus for instance

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Elementary Gates

Definition 3 Quantum bits:

- Pure state *of n qubits is $|x\rangle$ where $x \in \{0,1\}^n$. Pure states form an orthonormal basis of $\mathbb{C}^n$.*
- State *of a single qubit is $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$ where $\alpha_i \in \mathbb{C}$ such that $|\alpha_0|^2 + |\alpha_1|^2 = 1$.*
- State *of n qubits is $|\psi\rangle = \sum_{x\in\{0,1\}^n} \alpha_x|x\rangle$ where $\alpha_x \in \mathbb{C}$ such that $\sum_{x\in\{0,1\}^n} |\alpha_x|^2 = 1$.*

Definition 4 Gate *unitary matrix that acts on a small number of qubits (say at most three).*

Examples of single qubit gates:

- Bitflip $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- Phaseflip $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
- Phasegate $R_\varphi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$ (rotation phase by φ)
- Hadamard $H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$ we often say that $H|0\rangle = |+\rangle$ and $H|1\rangle = |-\rangle$.
- Controlled not $CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

Measurements

When we measure a vector $|\psi\rangle = \sum_{x\in\{0,1\}^n} \alpha_x|x\rangle$ we get the basis state $|x\rangle$ with probability $|\alpha_x|^2$ and the state collapses to $|x\rangle$ (becomes $|x\rangle$ and all other information is lost).

Quantum Teleportation, Quantum Circuits, and Early Algorithms

Einstein Podolsky Rosen (EPR) pair $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$.

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Algorithms *as part of series* Quantum computing

Grover algorithm

Algorithm for finding a solution for SAT instance.

Theorem 1 (Valiant, Vazirani) *There exists a probabilistic polynomial-time algorithm $\mathcal{A}$ such that for every Boolean formula φ with n variables:*

$$\varphi \in SAT \;\Rightarrow\; \Pr[\mathcal{A}(\varphi) \text{ has unique solution}] \geq \frac{1}{n} \text{ and}$$
$$\varphi \notin SAT \;\Rightarrow\; \Pr[\mathcal{A}(\varphi) \in SAT] = 0.$$

Theorem 2 *There is a quantum algorithm running in time poly$(n)2^{n/2}$ that given any circuit $C\colon \{0,1\}^n \to \{0,1\}$ of polynomial size finds $x \in \{0,1\}^n$ such that $\varphi(x)$ whenever x.*

Simon algorithm

Definition 3 (Simon's problem) *The input is a polynomial-size circuit for a function $f\colon \{0,1\}^n \to \{0,1\}^n$ such that there exists $a \in \{0,1\}^n$ satisfying:*

$$\forall x, y \in \{0,1\}^n : \quad f(x) = f(y) \;\Leftrightarrow\; x = y \oplus a.$$

Output is the string a.

Theorem 4 *There is a polynomial-time quantum algorithm for Simon's problem.*

Shor algorithm

Theorem 5 *There is a quantum algorithm that given a number N, runs in time polynomial in $\log(N)$ and outputs prime factorization of N.*

Bibliography

[1] Arora, Sanjeev, and Boaz Barak. *Computational complexity: a modern approach.* Cambridge University Press, 2009. Available online.

[2] Kitaev, A. Yu, A. H. Shen, and M. N. Vyalyi. *Classical and Quantum Computation* Graduate Studies in Mathematics vol 47 (Providence, RI: American Mathematical Society)." (2002).

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Standalone Talks

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Presented paper by Nathaniel Johnston

Non-uniqueness of minimal superpermutations

(http://www.sciencedirect.com/science/article/pii/S0012365X1300157X)

Introduction

We examine the open problem of finding the shortest string that contains each of the $n!$ permutations of n symbols as contiguous substrings (i.e., the shortest superpermutation on n symbols). It has been conjectured that the shortest superpermutation has length $\sum_{k=1}^{n} k!$ and that this string is unique up to relabelling of the symbols. We provide a construction of short superpermutations that shows that if the conjectured minimal length is true, then uniqueness fails for all $n \geq 5$. We will then also show a concise proof[1] of a lower bound on the length of the minimal superpermutation, thus disproving the forementioned conjecture.

Definitions

Definition 1 *A superstring is a string containing each string s_i from a given set $S = \{s_1, s_2, \ldots, s_m\}$.*

Definition 2 *A superpermutation on n symbols $[n] = \{1, 2, \ldots, n\}$ is a superstring where $S = S_n$ is the set of all permutations of n symbols.*

Results

Conjecture 3 *The minimal superpermutation on n symbols is unique up to relabelling of the symbols.*

Theorem 4 *There are at least $\prod_{k=1}^{n-4} (n-k-2)!^{k \cdot k!}$ distinct (up to relabelling) superpermutations on $[n]$ of length $\sum_{k=1}^{n} k!$*

Theorem 5 *[1] The length of the minimal superpermutation on n symbols is at least $n! + (n-1)! + (n-2)! + n - 3$.*

Bibliography

[1] Robin Houston, Jay Pantone, Vince Vatter: A lower bound on the length of the shortest superpattern, http://oeis.org/A180632/a180632.pdf

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Presented paper by Andrey Kupavskii, János Pach, and Gábor Tardos

Tilings with noncongruent triangles

(https://arxiv.org/abs/1711.04504v3)

Is it possible to tile the plane with pairwise noncongruent triangles of the same area and perimeter?

If a xyz tiangle has a fixed perimeter and fixed xy side then the non-fixed vertex must be on an ellipse with foci x and y. If a xyz triangle has a fixed area and fixed xy side then the third vertex must an exact distance from the xy line. Therefore, if two triangles of the same area and perimeter share a side, then they are congruent.

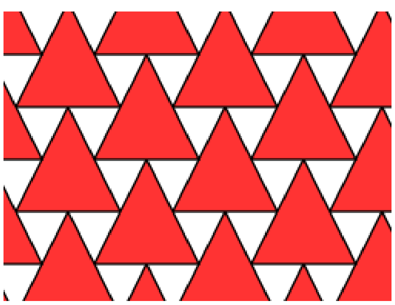
Theorem 6 *Let $k \geq 4$. In any tiling of a convex k-gon with finitely many triangles, there are two triangles that share an edge.*

There are counterexamples to polygon having only 3 sides (Fig. 1b) and when the number of tiling triangles can be locally infinite (Fig. 1c).

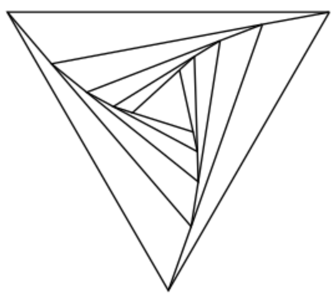
The following theorem implies that the answer to the initial question is NO.

Theorem 4 *Let $\mathcal{T}$ be tiling of the plane with triangles of unit perimeter, each of which has area at least $\varepsilon > 0$. Then there are two triangles in $\mathcal{T}$ that share a side.*

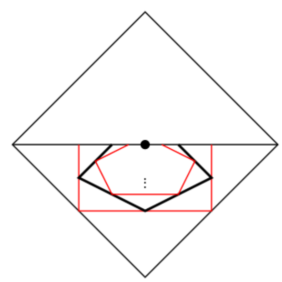
Theorem 5 *Let $\mathcal{T}$ be a locally finite tiling of the plane with triangles, and suppose that the lengths of their sides blong to interval $[1, 2)$. Then there are two triangles in $\mathcal{T}$ that share a side.*



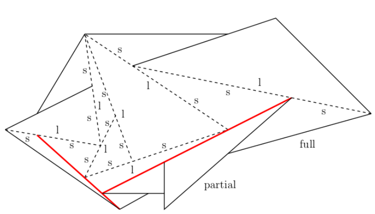
(a) Plane tiling with two equilateral triangle types.



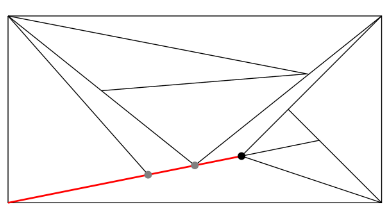
(b) Triangle tiling with no two triangles sharing a side.



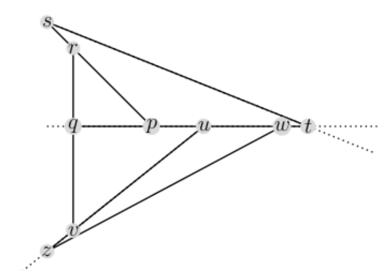
(c) Locally infinite tiling of a square, no two share a side.



(d) Part of a plane tiling



(e) Segments of an edge



(f) Proof illustration for Theorem 7

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Presented paper by Yoshihiro Asayama, Yuki Kawasaki, Seog-Jin Kim, Atsuhiro Nakamoto, Kenta Ozeki

3-dynamic coloring of planar triangulations

(https://www.sciencedirect.com/science/article/abs/pii/S0012365X18302425)

Introduction

An r-dynamic k-coloring of a graph G is a proper k-coloring such that any vertex v has at least $\min\{r, \deg_G(v)\}$ distinct colors in $N_G(v)$. The r-dynamic chromatic number $\chi_r^d(G)$ of a graph G is the least k such that there exists an r-dynamic k-coloring of G.
Loeb, Mahoney, Reiniger, and Wise [1] showed that if G is a planar graph, then $\chi_3^d(G) \leq 10$, and there is a planar graph G with $\chi_3^d(G) = 7$. Thus, finding an optimal upper bound on $\chi_3^d(G)$ for a planar graph G is a natural interesting problem.

Main result

In this paper, authors give a brief review of r-dynamic coloring of plane graphs. They focus on 3-dynamic chromatic number of planar triangulations and show the following result.

Theorem 1 *If G is a planar triangulation, then $\chi_3^d(G) \leq 5$. The upper bound is sharp.*

To show sharpness of the upper bound it suffices to consider graph of the octahedron, which is a planar triangulation and its 3-dynamic chromatic number equals 5.

In this talk, we will focus on proof of Theorem 1. The proof will be done by induction on the order. First we specify configurations appeared in planar triangulations G by discharging method. Then, we introduce some reductions to apply such configurations and obtain a smaller triangulation G'. Note that we do not apply the reductions if it destroys the simpleness, which guarantees that the minimum degree of G' is at least 3. Therefore by the induction hypothesis, there exists a 3-dynamic 5-coloring in G', which satisfies that every vertex in G' has distinct 3 colors in its neighborhood. This allows us to avoid the difficulty of reducibility arguments.

Lemma 2 *A planar triangulation has either a 3-vertex, a 5-vertex, a 4-4-edge, a 4-6-edge, or a 4-7-edge.*

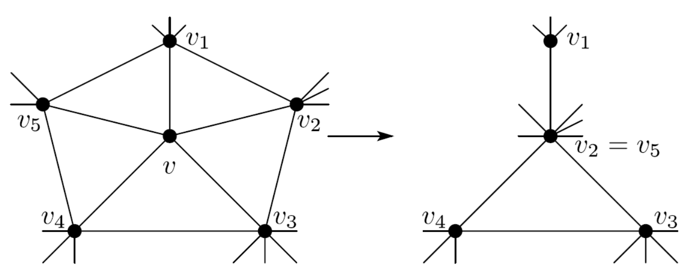

Figure 3: A 5-vertex contraction of v at $\{v_2, v_5\}$.

Lemma 3 *Let G be a planar triangulation with a 5-vertex, and H be the planar triangulation obtained from G by the 5-vertex contraction. If H is 3-dynamically 5-colorable, then so is G.*

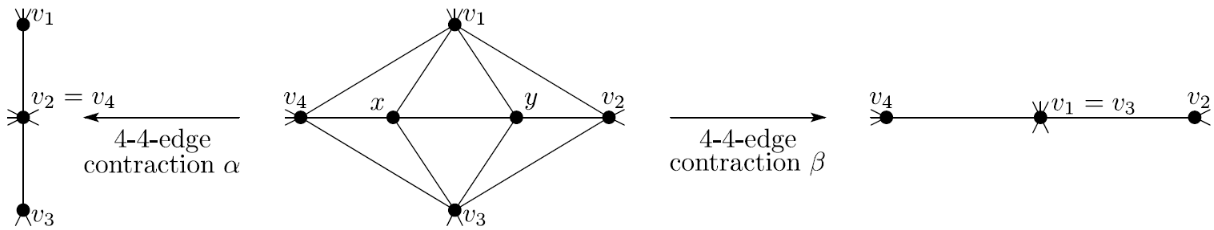

Figure 4: A 4-4-edge contraction α and β of xy.

Lemma 4 *Let G be a planar triangulation with a 4-4-edge, and H be the graph obtained from G by the 4-4-edge contraction α or β. If H is 3-dynamically 5-colorable, then so is G.*

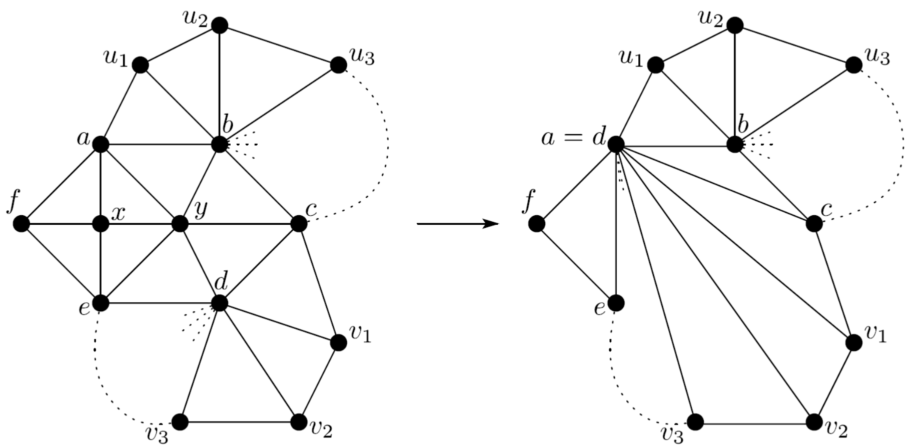

Figure 5: A 4-6-edge contraction of xy at $\{a, d\}$.

Lemma 5 *Let G be a planar triangulation with a 4-6-edge, and H be the graph obtained from G by the 4-6-edge contraction. If H is 3-dynamically 5-colorable, then so is G.*

Let G be a planar triangulation with 4-7-edge. In this case, we use an ordinary contraction of an edge in a triangulation.

Lemma 6 *Let G be a planar triangulation with a 4-7-edge, and H be the graph obtained from G by the 4-7-edge contraction. If H is 3-dynamically 5-colorable, then so is G.*

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Monge property for interval matrices

Introduction

A matrix $M \in \mathbb{R}^{m \times n}$ is said to have a Monge property if for every $i < k$ and $j < l$ it holds that

$$m_{ij} + m_{kl} \leq m_{il} + m_{kj}.$$

This property proved itself useful when approaching hard problems as the famous travelling salesman problem or the assignment problem. Although NP-hard in general, both problems are polynomially solvable when the cost matrix is Monge. For many other geometrical or optimization problems there are known algorithmical speed ups as well.
In this lecture we focus on Monge property for interval matrices. The study of interval analysis allows us to cope with problems of computational precision as well as data inaccuracy. In interval analysis we envelope our data into intervals and then perform calculations on these intervals instead of real values. The methods of interval analysis then ensure that the result is included in the resulting interval.
We present two definitions of interval Monge matrices, discuss their advantages and disadvantages as well as some of their properties. After that, we present some results about applications of interval Monge matrices in optimization problems.

Pavel Koblich Dvořák
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Communication Complexity Lower Bound of Disjointness

Introduction

In this talk I will present an elegant proof of communication lower bound of disjointness. This result was first proved by Kalyanasundaram and Schnitger [1], further simplified by Razborov [2] and Bar-Yossef et al. [3]. This talk follows the lecture notes of Radhakrishnan [4].

Communication Model

In the *communication model with private randomness* there is Alice and Bob and they know some boolean function $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$. Alice gets some $x \in \{0,1\}^n$ and Bob gets $y \in \{0,1\}^n$ and each of them gets some private random bits r_A, r_b respectively. Their task is to compute $f(x,y)$ with high probability. Let π be a protocol and $\pi_o(x,y)$ be an output of this protocol. The *randomized communication complexity* $R_\varepsilon(f)$ of the function f is the length of the optimal randomized protocol π such that for every $x, y \in \{0,1\}^n$ holds that $Pr_{r_A,r_B}[f(x,y) \neq \pi_o(x,y)] \leq \varepsilon$.

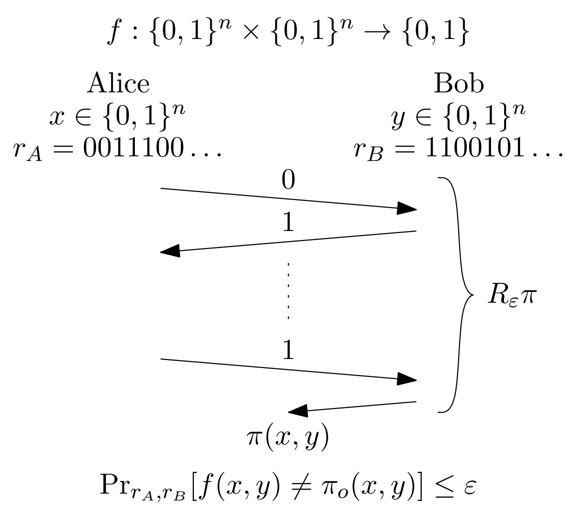

Figure 6: Communication model with private randomness.

The function Disjointness (Disj) is 1 if the input x and y are disjoint (we interpret x and y in $\{0,1\}^n$ as characteristic vectors of subsets of $[n]$).

Theorem 1 $R_{1/2-\varepsilon} \geq \Omega(\varepsilon^2 n)$.

Proof Idea

- We will prove that Alice or Bob have to send a non-trivial information about each bit of their input.
- From a short protocol π for Disj we derive a protocol τ for NAnd which reveal only small information about the input, where
$$\text{NAnd}(a,b) = \neg(a \wedge b), a, b \in \{0,1\}.$$
- We prove that such protocol τ can not exist.

Information Theory

Let X, Y and Z be random variables over same finite set S. Let $s \in S$ and $p_s = \Pr[X = s], q_s = \Pr[Y = s]$.

Definition 2 *The* entropy *of X is*

$$H(X) = \sum_{x \in S} p_x \log \frac{1}{p_x}.$$

Definition 3 *The* mutual information *of X and Y is $I(X : Y|Z) = H(X|Z) - H(X|YZ)$, where $H(X|Z) = \mathbb{E}_z H(X|Z = z)$.*

Lemma 4 (Chain rule) *For random variables $X_1, \ldots, X_n$ and Y holds that*

1. $H(X_1, \ldots, X_n) = \sum_i H(X_i|X_{<i})$.
2. $I(X_1, \ldots, X_n : Y) = \sum_i I(X_i : Y|X_{<i})$.

Definition 5 *The* Hellinger distance *of X and Y is*

$$h(X, Y) = \sqrt{\frac{1}{2} \sum_s \left(\sqrt{p_s} - \sqrt{q_s}\right)^2}.$$

Definition 6 *The* total variation *of X and Y is*

$$\Delta(X - Y) = \frac{1}{2} \sum_s |p_s - q_s|.$$

Lemma 7 (Hellinger vs. total variation)

$$\Delta(X - Y) \leq \sqrt{2} h(X, Y).$$

Lemma 8 (Hellinger vs. information) *Let a random variable B be distributed uniformly over $\{b_1, b_2\}$ and $\pi(B)$ be a function (possibly randomized) of B. Then,*

$$I\Big(B : \pi(B)\Big) \geq h^2\Big(\pi(b_1), \pi(b_2)\Big).$$

Lemma 9 (Cut and paste) *Let $\pi(x, y)$ denote the transcript of a randomized protocol of some communication problem on input (x, y). Then, for all x, x', y, y',*

$$h^2\Big(\pi(x, y'), \pi(x', y)\Big) = h^2\Big(\pi(x, y), \pi(x', y')\Big).$$

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Infinitude of Primes Using Formal Languages

(https://doi.org/10.1080/00029890.2018.1496761)

Introduction

In this talk, we show the infinitude of primes using formal (especially regular) languages.

Building blocks

Definition 1 (Regular expression) *The set of regular expressions* $RE(\Sigma)$ *is the smallest set closed under concatenation, union and Kleene star that contains* $\emptyset$ *and* $\{x\}\,\forall x \in \Sigma$.
An expression is said to be regular if it belongs to the set of regular expressions.

Definition 2 (Regular languages) *A language* $L \subseteq \Sigma^*$ *is regular if and only if it can be represented by a regular expression.*

Definition 3 (Distinguishing extension) *Given a language* $L \subseteq \Sigma^*$, *and a pair of strings* $x, y \in \Sigma^*$, *a distinguishing extension is a string* $z \in \Sigma^*$ *such that exactly one of the two strings* xz, yz *is a member of* L.

Theorem 4 (Myhill-Nerode) *Let* $x \equiv_L y$ *if there is no distingishing extension for* x *and* y *with respect to* L. *Then* L *is regular if and only if* $\equiv_L$ *induces finitely many equivalence classes.*

Results

Proposition 5 *For all* $n \in \mathbb{Z}^+$, *the language* $L_n = \{w \in \{a,b\}^* : |w|_a - |w|_b \text{ is divisible by } n\}$ *is regular.*

Proposition 6 *Let* $\mathbb{P}$ *be the set of all primes, and let* $L = \bigcup_{p \in \mathbb{P}} L_p$.
Then $L = \{w \in \{a,b\}^* : |w|_a - |w|_b \neq \pm 1\}$.

Proposition 7 *The language* $L = \{w \in \{a,b\}^* : |w|_a - |w|_b \neq \pm 1\}$ *is not regular.*

Corollary 8 *The set of all primes is infinite.*

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The 2CNF Boolean Formula Satisfiability Problem and the Linear Space Hypothesis

(https://arxiv.org/abs/1709.10453)

Introduction

As a P versus NP, *(Strong) Exponential time hypothesis ((S)ETH)* is extensively used in computer science. It appears in proofs of lower bounds of problems or optimality of algorithms.

For a k-SAT is no known algorithm which runs in $s^{o(n)}$, which motivates formulation of (S)ETH. Similary there is no known algorithm for 2-SAT which runs in $o(n)$ space. This gives us motivation for *Linear space hypothesis (LSH)*. We can use LSH as a conditional for showing space optimality of some algorithms and lower bounds on space complexity of some problems.

The talk is structured as follows. First I'll define (S)ETH and give some fast preview on usage of (S)ETH. Then, I'll define sublinear space complexity class and reductions family which not hinders sublinear space complexity class. Then we will see current limitations around 2-SAT problem. Lastly I'll define the Linear space hypothesis and show one of its application in lower bound on space complexity.

ETH, SETH

Let $s_d = \inf\{c \mid \text{d-SAT has } 2^{cn} \text{ algorithm}\}$. Let $s_\infty = lim_{d\to\infty} s_d$.

Hypothesis 1 ***Exponential time hypothesis (ETH).*** *$s_k > 0$ for $k \geq 3$.*

Exercise 2 *$s_2 = 0$.*

If the ETH is true, then it is not possible to find clique of size k neither independent set of size k in a time $n^{o(k)}$. Some other problems, such as graph k-colorability, finding Hamiltonian cycles can't be solved by algorithm better than c^n for constant $c > 1$.

Hypothesis 3 ***Strong Exponential time hypothesis (SETH).*** $s_\infty = 1$

SETH is used in parametrized complexity for proving lower bounds. For a graph of treewidth w, SETH implies that the optimal time algorithm for runs in time $(2 - o(1))^t n^{O(1)}$.

It can be also used for optimality of polytime algorithm. For example orthogonal vector problem or longest common subsequence of two vectors or edit distance of two strings can't be solved in time $n^{2-\varepsilon}$ for some $\varepsilon > 0$ if SETH holds.

Parameters, reductions and classes

Let have an input representation x. An $|x|$ is a length of representation of x. A **log-space size parameter** $m(x)$ for a problem P is a mapping Σ^* to $\mathbb{N}$ such that m must be computed using log space (there exists a Turing machine which for input x outputs $m(x)$ in unary using at most $c \log |x| + d$ space for constants $c, d > 0$) and there exists a polynomial p satisfying $m(x) \leq p(|x|)$ for all instances x of P.

Note 4 *For a graph x we have a parameter number of vertices $m_v(x)$ and a parameter number of edges $m_e(x)$.*

For a CNF formula x we have standard parameters a number of variables $m_v(x)$ and a number of clauses $m_c(x)$.

L is a class of problems which can be solved by a log space deterministic Turing machine. A class **NL** contains problems solvable by a logspace *non*deterministic Turing machine.

Definition 5 *Let have a log space parameter m, a constant $\varepsilon \in (0,1)$ and some polylog function l. A function f is* ***sublinear for a parameter*** *m if $f(x) = m(x)^\varepsilon l(|x|)$.*

Complexity class **PsubLIN** is a class of parametrized decision problems (P, m) solvable by deterministic Turing machine in polynomial time in $|x|$ and sublinear space for a parameter m.

Note 6 *We can easily see that* $\mathrm{L} \subseteq \mathrm{PsubLIN} \subseteq \mathrm{P}$. *We don't know if any of inclusions is proper.*

As for P we need logspace reductions, for a class PsubLIN we need even more restricted reduction. For that let's define four reductions.

Definition 7 *A parametrized problem (P_1, m_1) is* ***L-m-reducible*** *(logspace reducible) to a parametrized problem (P_2, m_2), denoted by $(P_1, m_1) \leq_m^L (P_2, m_2)$ if there is a logspace computable function f and two constants $k_1, k_2 > 0$ such that for any instance x of a problem (P_1, m_1)*

1. *$x \in P_1$ iff $f(x) \in P_2$ and*
2. *$m_2(f(x)) \leq m_1(x)^{k_1} + k_2$.*

For a ***short-L-m-reduction*** *($\leq_m^{sL}$) we just replace inequality 2. in previous defintion for an inequality (2') $m_2(f(x)) \leq k_1 m_1(x) + k_2$.*

Definition 8 ***Polynomial-time sub-linear-space reduction family (SLRF)*** *is Turring reducibilty using oracle machines. A parametrized problem (P_1, m_1) is* ***SLRF-T-reducible*** *to a parametrized problem (P_2, m_2), denoted by $(P_1, m_1) \leq_T^{SLRF} (P_2, m_2)$ if there exists $\varepsilon > 0$, an oracle Turing machine M^{P_2} with a write-only query tape, a polynomial p, a polylog function l, constants $k_1, k_2, k_3, k_4 > 0$ such that for every inst ace x of (P_1, m_1)*

1. *M^{P_2} runs in at most $p(|x|)$ time using at most $m_1(x)^\varepsilon l(|x|)$ space (query tape doesn't have to follow that restriction)*
2. *if M^{P_2} makes a query to P_2 with a word z on a query tape, then $m_2(z) \leq m_1(x)^{k_1} + k_2$ and $|z| \leq |x|^{k_3} + k_4$, and*
3. *after performing query to P_2 in a single step, M automatically erases the query tape and changes inner state according to answer of P_2 on z.*

We get the ***short-SLRF-T-reducibilty*** *($\leq_T^{sSLRF}$) by changing the first inequality in condition 2. above by an inequality $m_2(z) \leq k_1 m_1(x) + k_2$.*

For any reduction $\leq_r$, a problem (P, m) is $\leq_r$-complete for a given class $\mathcal{C}$ of problems if $P \in \mathcal{C}$ and every problem Q in $\mathcal{C}$ is $\leq_r$-reducible to P.

Fact 9 *Class* PsubLIN *is closed under $\leq_T^{sSLRF}$-reductions.*

There are problems X and Y such that $X \leq_T^{SLRF} Y$ and $X \not\leq_T^{sSLRF} Y$.

2-SAT and Linear space hypothesis

Definition 10 *2-SAT$_k$ is a collection of all 2-CNF formulas where each variable has at most k occurrences in whole formula.*

Proposition 11 *For each index $k \geq 3$ 2-SAT$_k$ is NL-complete.*

Theorem 12 *Theorem For a certain constant $c > 0$ and a polylog function l, 2-SAT with n variables and m clauses can be solved in polynomial time using $n^{1-c/\sqrt{\log n}} l(m+n)$ space.*

Lemma 13 *Lemma Let $m \in \{m_v, m_c\}$ and $k \geq 3$.*

- *(2-SAT$_k$,m)$\equiv_m^{sL}$(2-SAT$_3$,m) and*
- *(2-SAT$_3$,m_v)$\equiv_m^{sL}$(2-SAT$_3$,m_c).*

Hypothesis 14 ***Linear space hypothesis (LSH) for 2-SAT$_3$.*** *For any choice of $\varepsilon \in (0,1)$ and any polylog function l, no deterministic Turing machine solves 2-SAT$_3$ parametrized by m_v simultaneously in polynomial time using $m_v(x)^\varepsilon l(|x|)$ space, where x refers to an input instance to 2-SAT$_3$.*

Theorem 15 *If LSH for 2-SAT$_3$ holds, then*

- *L$\neq$NL,*
- *$\leq_T^{sSLRF}$(2SAT$_3$,m_v)$\nsubseteq$ PsubLIN,*
- *(2-SAT,m_v)$\notin$PsubLIN and*
- *there are two parametrized decision problems (P_1, m_1) and (P_2, m_2) in $\leq_T^{sSLRF}$(2-SAT$_3$,m_v) such that $(P_1, m_1) \nleq_T^{sSLRF} (P_2, m_2)$ and $(P_2, m_2) \nleq_T^{sSLRF} (P_1, m_1)$.*

Note 16 ***Alternative problem for LSH formulation.*** *For any directed graph $G = (V, E)$ of degree (indegree plus outdegree) at most k and two designated vertices s and t. Is there any path from s to t?*

Application of LSH as conditional

1NFA Search problem (Search-1NFA)

We have a 1NFA $M = (Q, \Sigma, \delta, q_0, F)$ with no λ-moves, and a parameter 1^n for $n \in \mathbb{N}$. Find a string x of the length n accepted by M (i.e. x is written on read-only tape of M and M enters a final state F before it reads whole input string x).

A log space parameter for this problem is $m_n(x) = |Q||\Sigma|n$.

Theorem 17 *Assuming LSH for 2-SAT$_3$, for every fixed value $\varepsilon \in (0, 1/2)$, there is no polytime O($n^{1/2-\varepsilon}$)-space algorithm for (Search-1NFA,m_n) problem.*

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Presented paper by Dimitriy Zakharov

Acute Sets

(https://arxiv.org/pdf/1707.04829.pdf)

Introduction

A set of points in R^d is *acute*, if any three points of this set form an acute triangle. In 1962 Danzer and Grünbaum [2] posed the following question: what is the maximum size $f(d)$ of an acute set in R^d? They proved a linear lower bound $f(d) \geq 2d-1$ and conjectured that this bound is tight. However, in 1983 Erdős and Füredi [3] disproved this conjecture in large dimensions. Through the time, there has been improvements in the lower bound on $f(d)$:

1983 Erdős and Füredi [3] prove $f(d) \geq 0.5 \cdot 1.154^d$. Their proof is a nonconstructive application of the probabilistic method.

2009 Ackerman and Ben-Zwi [1] prove $f(d) \geq c\sqrt{d} \cdot 1.154^d$, where c is a particular constant.

2011 Harangi [4] proves $f(d) \geq c \cdot 1.2^d$. He uses similar approach to Erdős and Füredi.

Two results

We show two improvements, both constructive. In the first theorem we prove $f(d+2) \geq f(d)$ which implies

Theorem 1 *There is an acute d-dimensional set witnessing* $f(d) \geq 2^{d/2} = 1.414^d$.

Second theorem is just stronger version of the firs one. The method in the first theorem is slightly improved to the statement that follows.

Theorem 2 *There is an acute d-dimensional set of size* F_{d+1}, *hence* $f(d) \geq F_{d+1} = 1.618^d$.

Here F_d is d-th Fibonacci number given by the first terms $F_1 = F_2 = 1$ and recurrence $F_{d+2} = F_{d+1} + F_d$. Both results are quite easy and uses a simple argument that we double some points when finding a particular witnessing set one/two dimensions higher.

In the first theorem we double all points such that both copies lies on a very small circle (in two new dimensions) around the latter point. In the second case, when searching for an acute set in R^d, we double the points on a particular half-plane with at least F_{d-1} points.

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Rank Estimators in Robust Regression—With an Application of Birkhoff's Theorem

Linear regression. Given data $X \in \mathbb{R}^{n\times p}$ and $y \in \mathbb{R}^n$, find $\beta \in \mathbb{R}^p$ such that

$$y \approx X\beta$$

Rank estimators. Find $\beta \in \mathbb{R}^p$ by solving

$$\min_{\beta\in\mathbb{R}^p} F(\beta) = \min_{\beta\in\mathbb{R}^p} \sum_{i=1}^{n} a_i^{\beta}(y_i - x_i^T\beta),$$

where $a_i^{\beta} \in \mathbb{R}$ are given scores and x_i^T rows of X.

Assumption. The scores a_i^{β} are monotone and do not depend on the residual values $y_i - x_i^T\beta$, but on the order only. That is, the scores for nondecreasingly sorted residuals are

$$\alpha_1 \leq \cdots \leq \alpha_n$$

Proposition 1 *Under Assumption, $F(\beta)$ is convex and piecewise linear and*

$$F(\beta) = \max_{\pi\in S_n} \sum_{i=1}^{n} \alpha_{\pi(i)}(y_i - x_i^T\beta).$$

Linear programing formulation. The problem

$$\min_{\beta\in\mathbb{R}^p} F(\beta) = \min_{\beta\in\mathbb{R}^p} \max_{\pi\in S_n} \sum_{i=1}^{n} \alpha_{\pi(i)}(y_i - x_i^T\beta)$$

has a linear programming formulation with $n!$ constraints

$$\min\ z;\ \ \sum_{i=1}^{n} \alpha_{\pi(i)}(y_i - x_i^T\beta) \leq z\ \forall\pi \in S_n$$

Problem statement. Given $\beta^* \in \mathbb{R}^p$, check whether it is optimal.

Lemma 2 *Optimality of β^* is equivalent to feasibility of*

$$\sum_{\pi\in S_n} \gamma_\pi \sum_{i=1}^{n} \alpha_{\pi(i)} x_i = 0, \quad \gamma_\pi \geq 0, \quad \sum_{\pi\in S_n} \gamma_\pi = 1.$$

Reminder.

- $B \in \mathbb{R}^{n\times n}$ is doubly stochastic if $B \geq 0$ and $\sum_{i=1}^{n} b_{ij} = \sum_{j=1}^{n} b_{ij} = 1$
- Birkhoff's theorem: Each doubly stochastic matrix is a convex combination of (at most $n^2 - 2n + 2$) permutation matrices

Theorem 3 *β^* is optimal iff $\exists G$, a positive multiple of a doubly stochastic matrix, such that*

$$0 = \sum_{i=1}^{n} x_i \sum_{j=1}^{n} \alpha_j G_{ij}.$$

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Computational complexity of crossing number problem

The *crossing number* $cr(G)$ of a graph G is the minimum number of pairwise edge crossings in a drawing of G in the plane. Determining the crossing number of a given graph G is a notoriously hard algorithmic problem in topological graph theory. This problem remains NP-hard even when the input graph G is obtained by adding one edge to a planar graph [1] (such G is called *almost-planar* or near-planar).

In this talk we would like to report on recent original results concerning the computational complexity of the crossing number problem. These results have been obtained in collaboration with Gelasio Salazar, Marek Derňár, and Carsten Thomassen. Namely, we survey the following:

I. *(ISAAC 2015, [2])* It is NP-hard to minimize the number of mutual edge crossings of two graphs which are simultaneously embedded into an orientable surface of genus 4 or more (actually, we improve from genus 6 in [2] to genus 4). This remains true even if the inputs are restricted to simple 3-connected graphs.

II. *(by-product of [2])* The (ordinary) crossing number problem remains NP-hard even if the input is restricted to almost-planar graphs having a bounded number, namely at most 8 (again improved from the note in [2]), vertices of degree greater than 3. This strengthens the result of [1].

III. *(SoCG 2016, [3])* The problem whether a graph G can be drawn with at most k crossings does not have a polynomial kernel in the parameter k. This means that, under complexity-theoretic assumptions, there is no polynomial-time algorithm that would transform the graph G into a (smaller) graph G_0 of size polynomial in k and associated k_0, such that $cr(G) \leq k$ if and only if $cr(G_0) \leq k_0$.

IV. *(SIDMA, [4])* It is NP-hard to determine whether the crossing number of a graph is even or odd. (Note that it is hence unlikely that this decision problem would be in NP.)

The unifying motif of these results is a careful exploitation of the hardness construction of almost-planar graphs by Cabello and Mohar [1]. In a very brief sketch, the original construction of [1] defines two planar graphs, "red" and "blue" one, both attached to a common rectangular "very thick frame". This gives a planar graph again, but then adding an edge between the red and the blue graph forces these two subgraphs to overlay each other, as sketched in Fig. 7. Determining the crossing number of the resulting graph is NP-hard.

To prove (I), we use the red and blue graphs of [1] as two separate graphs, and we add to them special gadgets which confine specified pieces of the red and the blue graphs to the same four handles of the surface. This, in turn, emulates the common frame of the original construction, and so it proves hardness of the joint crossing number problem (I). We can also combine parts of our new gadgets with the original frame in order to reduce the total number of vertices which have to be attached to the frame, implying (II).

In (III) and (IV), we use another modification of the construction, now concerning the crossing number of so-called *tiles*, which are graphs T drawn on a plane strip such that specified two vertices

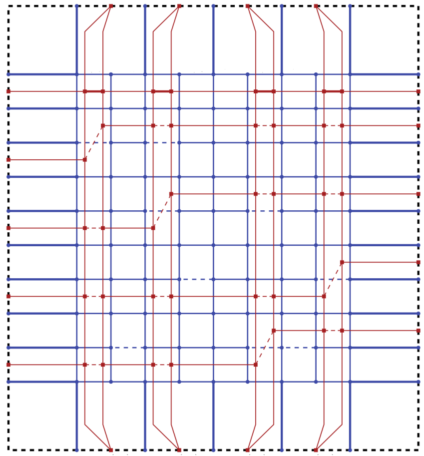

Figure 7: An illustration of the construction of an almost-planar graph taken from [1].

of T are fixed to the left boundary and another two specified vertices of T are fixed to the right boundary of the strip. The construction of [1] can also be interpreted such that we have a planar tile and twist one boundary of it, and then the crossing number problem of such a twisted planar tile is also NP-hard. A nice property of the latter problem is that, if we combine several instances of twisted planar tiles, the resulting crossing number can be the minimum of the crossing numbers of these instances. Additional arguments along this basic idea then prove each of (III) and (IV).

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Constant query time $(1+\varepsilon)$-approximate distance oracle for planar graphs

(https://arxiv.org/abs/1706.03108)

Introduction

A distance oracle is a data structure which keeps precomputed distance information and returns distance between any pair of the given vertices u, v efficiently. The efficiency is measured by the memory needed for the data structure (oracle size), the time for answering the query (query time) and the time for building the data structure for a given graph G (preprocessing time). In this talk we will be interested in oracles for planar graphs. For weighted planar digraphs it was proven in [2] that for any oracle size $S \in [n, n^2]$ there is an exact distance oracle with query time $O(\max\{1, \frac{(n^{1.5})}{S}\})$.

Definition 1 (α-approximate oracle) *The oracle is called (α)-approximate if it returns distance $\tilde{d}_G(u,v)$ s.t.*

$$d_G(u,v) \leq \tilde{d}_G(u,v) \leq d_G(u,v)$$

for every u, v in G.

In this article authors give $(1+\varepsilon)$-approximate oracle with $O(1)$ with nearly linear preprocessing time and nearly linear size.

Main result

Theorem 2 *Let G be an undirected planar graph with n vertices and non-negative weights and let $\varepsilon > 0$. There is $(1+\varepsilon)$-approx. distance oracle for G with $O(1)$ query time, $O(n \log n(\frac{logn}{\varepsilon} + f(\varepsilon)))$ size and $O(n \log n(\frac{\log^3 n}{\varepsilon^2} + f(\varepsilon)))$ where $f(\varepsilon) = 2^{O(\frac{1}{\varepsilon})}$.*

Definitions

Definition 3 (Shortest path separator) *A set $\mathcal{P}$ of shortest paths in graph G is a shortest path separator of G if $G[V(G)\backslash W]$, $W = \cup_{Q\in\mathcal{P}} V(Q)$, has at least $t \geq 2$ nonempty connected components $G_1, G_2, \ldots, G_t$.*

The separator is called α-balanced if for each G_i holds $|V(G_i)| \leq \alpha|V(G)|$.

Definition 4 (Boundary) *For a subgraph G_i of G, a set $B(G_i)$ of paths is a boundary of G_i if $B(G_i)$ separates G_i and the rest of G and for every path $Q \in B(G_i)$, there is an edge connecting Q and G_i.*

Definition 5 (α-balanced recursive subdivision) *α-balanced recursive subdivision is a structure which decomposes G into subgraphs $G_1, \ldots, G_t$ by α-balanced shortest path separator. Each G_i is then decomposed recursively until the subgraphs reach predefined size.*

Oracle with additive stretch

The recursive subdivision from the following lemma will be used in our oracle.

Lemma 6 *[3] Given a graph G and a shortest path spanning tree T_r of G, a $\frac{1}{2}$-balanced recursive subdivision T_G of G can be computed in $O(n \log n)$ time s.t. for each internal node X of T_G*

$|\mathcal{P}(X)| = O(1)$ *and for each node* X, $|B(X)| = O(1)$*. Moreover, for each node* X *of* T_G *and each root path* Q *of* T_r*, if* $Q \in B(X)$*, then* $Q \in \mathcal{P}(X')$ *for some ancestor* X' *of* X *in* T_G*.*

Lemma 7 *[4] For a path* Q *in* G*,* $\varepsilon > 0$ *and* $D \geq d(Q)$*, a set* P_Q *of* $O(\frac{1}{\varepsilon})$ *vertices in* $V(Q)$ *can be selected in* $O(|V(Q)|)$ *time such that for any pair of vertices* u *and* v *shortest-separated by* Q*,* $d_G(u,v) \leq \min_{p \in P_Q} d_G(u,p) + d_G(p,v) \leq d_G(u,v) + \varepsilon D$*.*

Theorem 8 *For graph* G *and* $\varepsilon_0 > 0$*, there is an oracle which gives a distance* $\tilde{d}(u,v)$ *with* $d_G(u,v) \leq \tilde{d}(u,v) \leq d_G(u,v) + 7\varepsilon_0 d(G)$ *for any vertices u,v in* $O(1)$ *query time,* $O(n(\log n/\varepsilon_0 + f(\varepsilon_0)))$ *size and* $O(n(\log^3 n/\varepsilon_0^2 + f(\varepsilon_0)))$ *preprocessing time.*

From the theorem above for $\varepsilon_0 = \frac{\varepsilon}{7c}$ for graph G s.t. $d_G(u,v) \geq \frac{d(G)}{c}$ for every two vertices u, v we have oracle from the main theorem. For other graphs G we will use the scaling technique from [1].

Scaling

Lemma 9 *[5] For* G *and* $\gamma \geq 1$*, connected subgraphs* $G(\gamma,1),\ldots,G(\gamma,n_\gamma)$ *of* G *with the following properties can be computed in* $O(n \log n)$ *time:*

1. *For each vertex* u *in* G*, there is at least one* $G(\gamma,i)$ *that contains* u *and every* v *with* $d_G(u,v) \leq \gamma$*.*
2. *Each vertex* u *in* G *is contained in at most* 18 *subgraphs.*
3. *Each subgraph* $G(\gamma,i)$ *has radius* $r(G(\gamma,i)) \leq 24\gamma - 8$*.*

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Sharing a pizza: bisecting masses with two cuts

(https://pdfs.semanticscholar.org/71d6/ae1c5324191a7874ad124e065c9cc08aa728.pdf)

Introduction

We will be interested in the problem of halving a pizza so that two people could have the same amount of each ingredient. The most classical result in the topic is the "ham sandwich" theorem which claims that for any $d \in \mathbb{N}$, any d measurable subsets of $\mathbb{R}^d$ with finite measure can be bisected with a single $(d-1)$-dimensional hyperplane. In particular, for a pizza (which we can consider as a 2-dimensional object) we are allowed to have only one type of topping on it if we want to bisect it with a single cut. In order to halve a pizza with more toppings, we need to either cut along some more complicated curve or use more cuts. We will focus on the latter and in particular we will show that two more toppings may be added if we are allowed to use two cuts.

Preliminaries

Definition 1 *A mass distribution μ on $\mathbb{R}^d$ is a finite measure on $\mathbb{R}^d$ (i.e. $\mu(\mathbb{R}^d) < \infty$) such that all open subsets are μ-measurable and $\mu(S) = 0$ for every lower-dimensional subset S of $\mathbb{R}^d$.*

Definition 2 *Let $\mathcal{L}$ be a finite set of oriented hyperplanes (i.e. for each $l \in \mathcal{L}$ we have two half-spaces, the negative half-space l^- and the positive half-space l^+). Define $\lambda(p) = |\{l \in \mathcal{L} : p \in l^+\}|$. Denote by R^+ the set $\{p \in \mathbb{R}^d : \lambda(p)$ is even$\}$ and by R^- the set $\{p \in \mathbb{R}^d : \lambda(p)$ is odd$\}$.*

Therefore, any set $\mathcal{L}$ induces a partition $\mathbb{R}^d = R^+ \cup R^-$.

Definition 3 *For any mass distributions $\mu_1, \ldots, \mu_n$ and any finite set $\mathcal{L}$ of oriented hyperplanes, we say that $\mathcal{L}$ simultaneously bisects $\mu_1, \ldots, \mu_n$ if $\mu_i(R^+) = \mu_i(R^-)$ for any $1 \leq i \leq n$.*

Main results

Theorem 4 *Let $\mu_1, \ldots, \mu_4$ be mass distributions in $\mathbb{R}^2$. Then there exist two lines l_1, l_2 such that $\{l_1, l_2\}$ simultaneously bisects $\mu_1, \ldots, \mu_4$.*

Theorem 5 *Let $\mu_1, \ldots, \mu_5$ be mass distributions in $\mathbb{R}^3$. Then there exist two planes l_1, l_2 such that $\{l_1, l_2\}$ simultaneously bisects $\mu_1, \ldots, \mu_5$.*

One may also impose some restrictions on the lines, but at the cost of reducing the number of mass distributions.

All the proofs rely on the Borsuk-Ulam theorem:

Theorem 6 (Borsuk-Ulam) *For any continuous function $f : \mathbb{S}^{k-1} \to \mathbb{R}^k$ there exists a point $p \in \mathbb{S}^{k-1}$ such that $f(p) = f(-p)$.*

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Going Far From Degeneracy

(https://arxiv.org/abs/1902.02526)

Introduction

In 1959, Erdős and Galai showed that every graph of degeneracy at least d contains a cycle of length at least $d+1$. On the other hand, the decision whether a graph contains a cycle of length at least $d+2$ is trivially NP-complete. In this paper, the authors give an algorithm that finds a cycle of length $d+k$ in time $2^{\mathcal{O}(k)}|V(G)|^{\mathcal{O}(1)}$ for 2-connected graphs. They also derive a similar algorithm for a path of length $d+k$ for connected graphs. For path the natural NP-completeness barrier is a decision about $d+2$-long path on general graphs. Moreover, the provided results are in a sense optimal since finding a $(1+\varepsilon)d$-long path (cycle) is NP-complete for connected (2-connected) graphs.

Since both results are quite similar, only the one for cycles will be presented.

Tools

The main tool is to derive the definition of segments (Definition 1) and then to show an equivalency with the original problem (See Figure 8). The advantage of this definition is that it is more suitable for standart FPTtechniques, such as color coding. Using this technique we derive the promissed algorithm.

Definition 1 (System of T-segments) *We say that a set $\{P_1, \dots, P_r\}$ of paths in G is a* system of T-segments *if it satisfies the following conditions.*

(i) For each $i \in \{1, \dots, r\}$, P_i is a two-terminal T-segment (See Definition 2),

(ii) $P_1, \dots, P_r$ are internally vertex-disjoint, and

(iii) the union of $P_1, \dots, P_r$ is a linear forest.

Definition 2 (Two-Terminal segments) *P is a* two-terminal T-segment *if it has at least three vertices, both end-vertices of P are in T and internal vertices of P are not in T.*

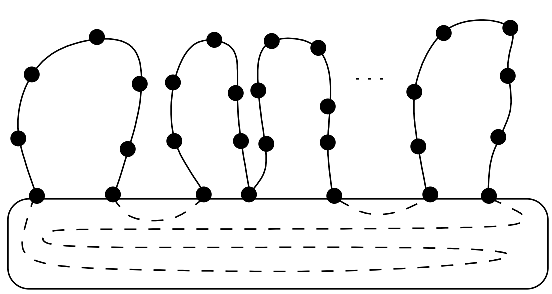

Figure 8: Basic idea of the equivalence of path and segments.

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Presented paper by Ken-ichi Kawarabayashi and Benjamin Rossman

A Polynomial Excluded-Minor Approximation of Treedepth

(http://www.math.toronto.edu/rossman/treedepth.pdf)

Introduction

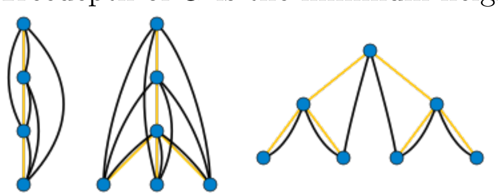
In this talk we are going to study notion of treedepth of simple graphs and obstructions for having small treedepth. There is a famous result that if a graph G has treewidth at least k^c, where c is some constant, then it contains $k \times k$ grid as a minor. In this talk we are going to prove result of similar flavor, but for the notion of treedepth. Namely that if a graph G has treedepth at least k^c, where c is some constant, then it contains either $k \times k$ grid as a minor or complete binary tree of height k as a minor or path on 2^k vertices.

Preliminaries

$G \preceq H$ means that G is a minor of H, i.e. G can be obtained from H by a sequence of vertices and edges deletions and edge contractions.
If T is a rooted tree then by $u \leq_T w$ we denote that w is an ancestor of u in T.

The treedepth of a graph G, denoted by $td(G)$, can be defined in many equivalent ways.

1. Treedepth of G is the minimum height of a forest F so that $G \subseteq Clos(F)$.



2. Treedepth of G is the minimum number of colors that need to be used to color vertices, so that for any connected subgraph H of G, it contains a vertex with unique color within this subgraph.
3. $$td(G) = \begin{cases} 1 & \text{if } |G| = 1 \\ \min_{v \in V(G)} 1 + td(G \setminus v) & \text{if } G \text{ is connected and } |G| > 1 \\ \max_{C \in cc(G)} td(C) & \text{otherwise} \end{cases}$$

A tree decomposition of a graph G is a pair $(T, \mathcal{W})$ where T is a tree and $\mathcal{W} = \{W_t\}_{t \in V(T)}$ is a family of sets $W_t \subseteq V(G)$ such that:

- $\bigcup_{t \in V(T)} = V(G)$, and every edge of G has both ends in some W_t,
- for every $v \in V(G)$, sets W_t containing v form a connected subtree of T.

Width of a tree decomposition $(T, \mathcal{W})$ is defined as $\max_{t \in V(T)} |W_t| - 1$ and a treewidth of G, denoted $tw(G)$, is the minimum width of a tree decomposition of G.

Main Theorem

There is a constant C such that every graph G with treedepth $\geq Ck^5 \log^2 k$ satisfies one or more of the following conditions:

- G has treewidth $\geq k$,
- G has the complete binary tree of height k as a minor,
- G contains a path of order 2^k.

Various lemmas along the way

1. $td(G) \leq \log_2(|V(G)|)$
2. If $(T, \mathcal{W})$ is a width-w tree decomposition of a graph G, then $td(G) \leq (w+1) \cdot td(T)$ (in particular $tw(G) + 1 \leq td(G) \leq (tw(G) + 1) \cdot \log n$)
3. $td(P_k) = \lceil \log(k + 1) \rceil$, where P_k is a path on k vertices
4. $td(B_h) = h$, where B_h is a full binary tree with height h
5. For every rooted tree T and $h \geq 0$ and $k \geq 1$, if $B_h \not\preceq_{rooted} T$ and $P_k \not\preceq T$, then $td(G) \leq h \cdot (\lceil \log(k + 1) \rceil + 1)$.
6. Every tree with treedepth $\geq d$ contains a subcubic (with degrees ≤ 3) subtree of order $\geq 2^{\sqrt{d}-2}$.
7. Let $h, c \geq 1$ and suppose G is a connected graph with maximum degree $\leq c$ such that $B_h \not\preceq G$ and $P_{2^h} \not\preceq G$. Then $|V(G)| \leq c^{O(h^2)}$.

Concluding proof

Ultimate goal: Every graph G contains a path of order 2^h or has a B_h−minor where

$$h = \Omega\left(\frac{r^{\frac{1}{4}}}{\log^{\frac{1}{2}}(tw(G) + 1)}\right), \; r = \frac{td(G)}{tw(G) + 1}.$$

Definiton (Greedy rooted tree decomposition)

- A greedy rooted tree decomposition of a connected graph G is a rooted tree T with the following properties:
  1. $V(T) = V(G)$
  2. $G \subseteq Clos(T)$
  3. for every child-parent pair $xy \in E(T)$, there exists $w \leq_T x$ such that $\{w, y\} \in E(G)$
- For each $x \in V(G)$, we define $Bag_{T,G} \subseteq V(G)$ by $Bag_{T,G}(x) \coloneqq \{x\} \cup \{y : \text{there exists } w \text{ such that } w \leq_T x <_T y \text{ and } \{w, y\} \in E(G)\}$.

Given a graph G we fix a greedy rooted tree decomposition T of width $tw(G)$ for G and we know that $td(T) \geq r$. Then we construct sequence of three trees:

1. Specific spanning tree $F \subseteq G$.
2. Subcubic rooted subtree $S \subseteq T$ of order $|V(S)| = 2^{\Omega\sqrt{r}}$.
3. Subtree $Q \subseteq F \subseteq G$ with maximum degree $\leq tw(G) + 2$ and $V(S) \subseteq V(Q)$.

We conclude by observing that Q contains a path of length 2^h or a B_h-minor where h is as stated before.

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A bound on the inducibility of cycles

(https://arxiv.org/abs/1801.01556)

Introduction

In 1975, Pippenger and Golumbic conjectured that every n-vertex graph has at most $n^k/(k^k - k)$ induced cycles of length $k \geq 5$. In the paper, the new upperbound $2n^k/k^k$ is proven.

The result

The *induced density* of a graph H in a graph G, denoted by $i(H, G)$, is the number of induced copies of H in G divided by $\binom{|V(G)|}{|V(H)|}$. The *inducibility* of a graph H, denoted by $ind(H)$, is the limit of the sequence $i(H, n)$ where $i(H, n)$ is the maximum induced density of H in an n-vertex graph.

Let G is an n-vertex graph and $(z_1, z_2, z_3, \ldots, z_k)$ is a k-tuple of vertices of G. We say that the k-tuple is *good* if $z_2z_1z_3z_4 \cdots z_k$ is an induced cycle of length k in G.

Theorem 1 *Every n-vertex graph G contains at most $2n^k/k^k$ induced copies of a cycle C_k of length $k \geq 5$.*

Definition 2 *Let G be a graph and $D = (z_1, z_2, z_3, \ldots, z_k)$ be a good k-tuple. We define a* weight *$w(D)$ of D as*

$$w(D) = \prod_{i=1}^{k} \frac{1}{n_i} ,$$

where

- *n_1 is n,*
- *n_2 is the number of neighbors of z_1,*
- *n_3 is the number of neighbors of z_1 that are not neighbors of z_2,*
- *n_i for $i = 4, \ldots, k-1$ is the number of vertices x such that $z_2z_1z_3z_4 \cdots z_{i-1}x$ is an induced path of length i, and*
- *n_k is the number of vertices x such that $z_2z_1z_3z_4 \cdots z_{k-1}x$ is an induced cycle of length k.*

Note 3 (AM-GM inequality) *Every nonnegative real numbers $x_1, x_2, \ldots, x_k$ satisfy:*

$$\Big(\prod_{i=1}^{k} x_i\Big)^{\frac{1}{k}} \leq \frac{x_1 + x_2 + \cdots + x_k}{k}$$

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Presented paper by Asahi Takaoka

A vertex ordering characterization of simple-triangle graphs

(https://arxiv.org/pdf/1611.09003.pdf)

Introduction

For a family S of sets, we can define its intersection graph, whose vertices are in one-to-one correspondence to elements from S, and two vertices are adjacent if and only if their corresponding sets intersect. In such a way we can define many graph classes, depending on the objects we consider as elements of S. For example, interval graphs are intersection graphs of intervals on the real line, and unit disk graphs are intersection graphs of unit radius disks on the plane. In the talk we introduce the class of simple-triangle graphs and present a characterization of simple-triangle graphs in terms of certain vertex orderings

Definitions

Let L_1 and L_2 be two horizontal lines on the plane. A *triangle* is a pair (p, I), such that p is a point which belongs to L_1 and I is an interval contained in L_2. The interval I is the *base* and the point p is the *apex* of the triangle. A graph G which is an intersection graph of set S of such triangles is called a *simple-triangle graph* and S is called the *representation* of G.

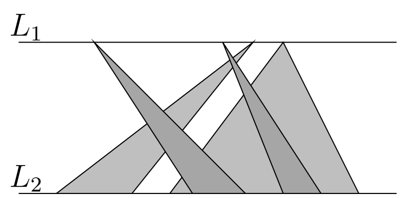

Figure 9: A cycle C_4 is a simple-triangle graph.

A pair $P = (V, \prec_P)$, where V is a finite set and $\prec_P \subseteq V \times V$ is a *partial order* if $\prec_P$ is irreflexive and transitive. It is a *linear order* if it is partial and for every $u, v \in V$ it holds that $u \prec_P v$ or $v \prec_P u$. It is an *interval order* if it is partial and for each $v \in V$ there exists an interval $I(v) = [l(v), r(v)]$ on the real line such that $u \prec_P v$ if and only if $r(u) < l(v)$. It is an *linear-interval order* if there exist a linear order $L = (V, \prec_L)$ and an interval order $P_I = (V, \prec_{P_I})$ such that $P = L \cap P_I$ (i.e $u \prec_P v \Leftrightarrow u \prec_{P_I} v$ and $u \prec_L v$). A pair $L = (V, \prec_L)$ is a *linear extension* of partial order $P = (V, \prec_P)$ if it is a linear order and if $u \prec_L v$ whenever $u \prec_P v$ for each $u, v \in V$.

An orientation of a graph G is *acyclic* if it has no directed cycle. It is *transitive* if for any $u, v, w \in V(G)$ if $u \to v$ and $v \to w$ then $u \to w$. It is *alternating* if it is transitive on every induced subgraph C_n of G, for $n \geq 4$.

A graph G is a *comparability graph* if and only if G has a vertex ordering σ such that for any $u \prec_\sigma v \prec_\sigma w$ if $uv, vw \in E(G)$, then $uw \in E(G)$. Equivalently, it has a vertex ordering that contains no subordering shown in Fig. 10 (b). A complement $\overline{G}$ of comparability graph G is called a *cocomparability graph*.

A vertex ordering σ of G satisfies the C_4 rule if for every induced $C_4 = (u, v, w, x)$ in G it holds

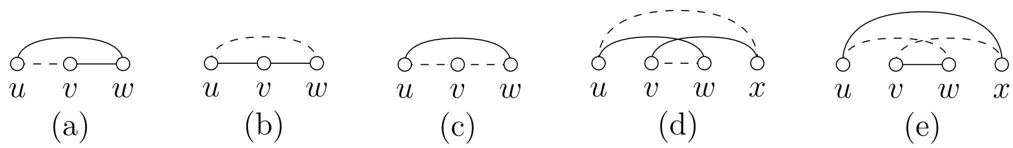

Figure 10: Forbidden patterns

that $u \prec_\sigma v \Leftrightarrow w \prec_\sigma v \Leftrightarrow w \prec_\sigma x \Leftrightarrow u \prec_\sigma x$. It satisfies the $2K_2$ rule if for every induced $2K_2 = \{uw, vx\}$ in G it holds that $u \prec_\sigma v \Leftrightarrow w \prec_\sigma v \Leftrightarrow w \prec_\sigma x \Leftrightarrow u \prec_\sigma x$.

Theorems

Theorem 1 *For a graph G the following conditions are equivalent:*

1. *G is a simple-triangle graph,*
2. *G has a cocomparability ordering fulfilling the C_4 rule,*
3. *G has a vertex ordering that contains no subordering shown in Fig. 10 (c),(d),(e),*
4. *$\overline{G}$ has a comparability ordering fulfilling the $2K_2$ rule,*
5. *$\overline{G}$ has a vertex ordering that contains no subordering shown in Fig. 10 (b),(d),(e).*

Theorem 2 *A graph G is a simple-triangle graph if and only if there exists an alternating orientation of G and a transitive orientation of $\overline{G}$ such that the union of the oriented edges of G and $\overline{G}$ forms an acyclic orientation of the complete graph.*

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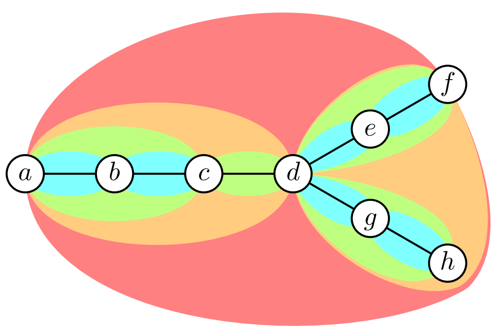
Top Tree Data Structure

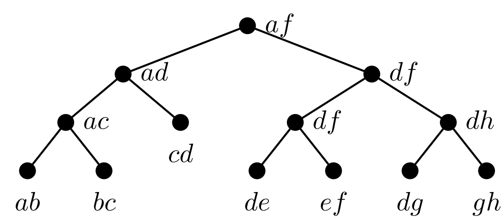
Introduction

The top tree data structure maintains a forest with some other data on paths or in individual trees. It provides methods for adding and removing edges and for accessing the data, all of them with logarithmic time complexity. Among similarly described data structures, top tree is one of the most general ones. Examples of stored data include: lengths of paths; minimal weight of edge on a path allowing decreasing all weights on the path by the same amount; tree diameter, median, or center.

Hierarchical clusterization

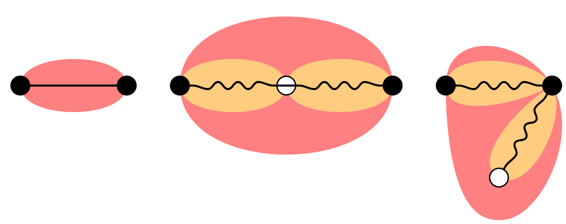
The data structure is based on a hierarchical partitioning of the sets of edges of individual *underlying* trees into so called *clusters*. Each cluster is a subtree with just two *boundary* vertices, which can be shared with neighbouring clusters; either it is an edge or it consists of two subclusters. The hierarchy of clusterization (in the left image) is stored as a binary top tree (in the right image):





Types of clusters (from left to right in the following image):

- *base* cluster (just an edge),
- *compress* cluster,
- *rake* cluster.



User interface

User data are associated with clusters. We have direct access to data in root clusters and we are supposed to update the data when the top tree structure is being changed; user-defined routines are called in such a case.

Methods:

- `link` (adds an edge),
- `cut` (removes an edge),
- `expose` (changes the boundary vertices of a root cluster),
- `non-local_search` (binary-like search preserving non-local properties).

User-defined routines:

- `split` (when root cluster is being destroyed making root clusters from its children),
- `destroy` (when base root cluster is being destroyed),
- `create` (when base cluster is being created),
- `join` (when cluster is being created out of two root clusters).

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Large feedback arc sets, high minimum degree subgraphs, and long cycles in Eulerian digraphs

(https://arxiv.org/pdf/1202.2602.pdf)

Introduction

A *minimum feedback arc set* of a directed graph (digraph) G is a smallest set of arcs whose removal makes G acyclic. We show lower bound of the cardinality which is denoted by $\beta(G)$. Using this result we prove an upper bound of length of cycles contained in an Eulerian digraph.

Notation

$\beta(G)$ . . . the minimum size of a feedback arc set
$g(G)$ . . . the length of the shortest cycle in G
cut . . . a partition of the vertices of a digraph into two disjoint subsets
2-cycle-free . . . between any pair of vertices, there do not exists arcs in two different directions
S . . . set of short arcs, its length is at most $\frac{n}{2}$
L . . . set of long arcs, its length is grater than $\frac{n}{2}$
s_i . . . for a vertex v_i is s_i the number of short arcs connecting v_i with some v_j where $j > i$
t_i . . . the number of long arcs of length i
A_i . . . set of vertices $\{v_1, \dots, v_i\}$
C_i . . . cut $(A_i, V \setminus A_i)$
c_i . . . the number of arcs crossing cut C_i, note $c_n = 0$
$F(s_1, \dots, s_n; t_1, \dots, t_n) := \sum_{i=1}^{n} \binom{s_i+1}{2} + (n-i)t_i$
$F(m, n) := \min F(s_1, \dots, s_n; t_1, \dots, t_n))$

Theorems

Theorem 1 *Every Eulerian digraph G with n vertices and m arcs has* $\beta(G) \geq \frac{m^2}{2n^2} + \frac{m}{2n}$.

Lemma 2 *In any cut* $(A, A \setminus V)$ *of an Eulerian digraph, the number of arcs from A to* $V \setminus A$ *equals the number of arcs from* $V \setminus A$ *to A.*

Lemma 3 $F(m, n) = tm - (t^2 - t)\frac{n}{2}$, *where* $t = \lceil \frac{m}{n} \rceil$.

Corollary 4 *Every Eulerian digraph G with n vertices and m arcs has* $g(G) \leq \frac{6n^2}{m}$.

Theorem 5 *Every Eulerian digraph G with n vertices and m arcs has an Eulerian subgraph with minimum degree at least* $\frac{m^2}{24n^3}$.

Proposition 6 *Every Eulerian digraph with n vertices and m arcs has a cycle of length at least* $1 + \lfloor \sqrt{\frac{m}{n}} \rfloor$.

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Presented paper by János Pach, Gábor Tardos

Tiling the plane with equilateral triangles

(https://arxiv.org/pdf/1805.08840.pdf)

Introduction

A collection of compact convex subsets of $\mathbb{R}^2$ is said to *tile the plane* if no two of them share an interior point and their union is equal to $\mathbb{R}^2$. In this article authors focus on tiling the plane with equilateral triangles such that no two of them share a side and the side lengths of the triangles are bounded from below by a positive constant. The main result is the following Theorem.

Theorem 1 *Let $\mathcal{T}$ be a tiling of the plane with equilateral triangles such that the side lengths of the triangles are bounded from below by a positive constant and no two triangles share a side. Then the triangles in $\mathcal{T}$ have at most three different side lengths, a, b and c with $a = b + c$ (where b may be equal to c) and the tiling is periodic.*

Definitions

Let $\mathcal{T}$ be a fixed tiling of the plane with equilateral triangles satisfying requirements of Theorem 1.

Definition 2 *An edge of a triangle $T \in \mathcal{T}$ is subdivided if some interior point of this edge is a vertex of another triangle $T' \in \mathcal{T}$. Otherwise, it is called uncut.*

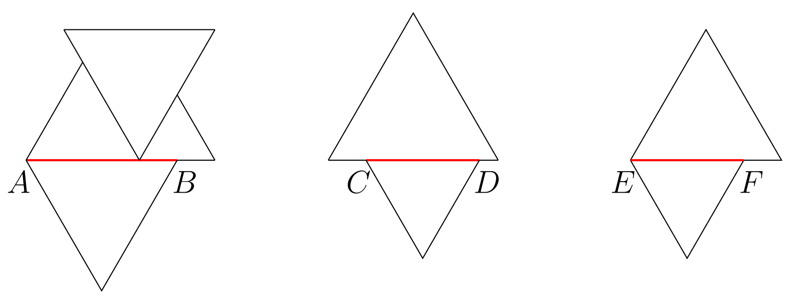

Figure 11: AB is subdivided, CD and EF are uncut.

Definition 3 *The edge AB continues at A if A is the interior point of an edge e of another triangle of the tiling and e also contains some interior points of the edge AB.*

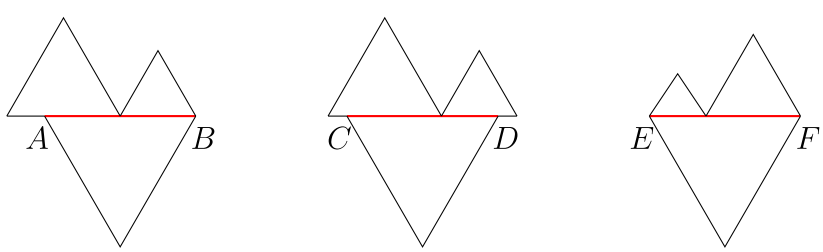

Figure 12: Edge AB continues at A but does not continue at B, edge CD continues at both C and D, and edge EF does not continue either at E and F.

Definition 4 *A triangle of the tiling is called:*
small – if all three of its sides are uncut;
large – if all three sides are subdivided;
improper – if it has an uncut side and also a side that does not continue in either direction.

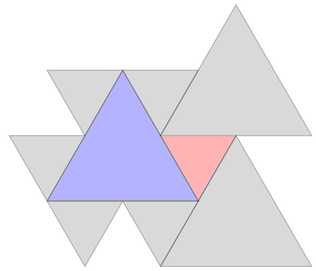

Figure 13: The blue triangle is large and the red triangle is small.

Lemmas

The proof Theorem 1 is based on the following lemmas.

Lemma 5 *If an edge AB is subdivided and does not continue at A, then there is a unique triangle ADE in the tiling such that D is an interior point of AB. The edge AD is uncut.*

Lemma 6 *For any improper triangle T in $\mathcal{T}$ there are two other triangles, $U, V \in \mathcal{T}$ such that U is improper and the sum of the side lengths of U and V does not exceed the side length of T.*

Lemma 7 *There is no improper triangle in the tiling.*

Lemma 8 *If the edge AB of a triangle ABC in $\mathcal{T}$ continues at A, then BC is uncut.*

Lemma 9 *Every triangle in $\mathcal{T}$ is either large or small. The sides of the large triangles do not continue in either direction, and each of them contains in its interior precisely one vertex.*

Lemma 10 *Let $T = ABC$ be a large triangle in the tiling and T_1, T_2 and T_3 be the large triangles containing A, B and C, respectively, in the interior of one of their edges. Then the side length of T is the average of the side lengths of T_1, T_2 and T_3.*

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Presented paper by Marcelo Aguiar and Federico Ardila

Hopf monoids and generalized permutahedra

(http://arxiv.org/abs/1709.07504)

Introduction

Many combinatorial structures provide intuitive "merging" and "breaking" operations (for example for graphs we can think of disjoint unions, induced subgraphs or contractions). Hopf monoids are an algebraic structure reflecting these operations as product and coproduct. Besides the algebraic point of view the talk will present a geometric perspective by introducing generalized permutahedra. Connecting these two perspectives turns out to be fruitful for finding nice antipode formulas and, consequently, unifying already known reciprocity results as well as developing new ones. All used notions and concepts will be explained during the talk.

Hopf monoids

Definition 1 *A* ***set species*** P *consists of*

a) for each finite set I, a set $P[I]$
b) for each bijection $\sigma : I \to J$, a map $P[\sigma] : P[I] \to P[J]$, s.t. $P[\sigma \circ \tau] = P[\sigma] \circ P[\tau]$ and $P[\mathrm{id}] = \mathrm{id}$

Definition 2 *A* ***connected Hopf monoid in set species*** H *consists of:*

a) a set species H *s.t.* $\mathrm{H}[\emptyset]$ *is a singleton (connectedness)*
b) product and coproduct maps

$$\mathrm{H}[S] \times \mathrm{H}[T] \xrightarrow{\mu_{S,T}} \mathrm{H}[I] \quad \textit{and} \quad \mathrm{H}[I] \xrightarrow{\Delta_{S,T}} \mathrm{H}[S] \times \mathrm{H}[T]$$

for each finite set I and decomposition $I = S \sqcup T$.

Moreover, the (co)product maps should satisfy 4 ***axioms*** *below.*

Axioms:

a) (Co)Associativity: product and coproduct are associative
b) Compatibility: "merging then breaking" = "breaking then merging"
c) Naturality: "relabeling maps" respect merging and breaking
d) Unitality: merging and breaking is trivial, whenever the underlying decomposition is trivial

Hopf monoid structures can be defined on (hyper-)graphs, matroids, posets, set partitions, paths, ...

Fix a field $\Bbbk$.

Definition 3 *A* ***vector species*** $\boldsymbol{P}$ *consists of:*

a) for each finite set I, a vector space $\boldsymbol{P}[I]$
b) for each bijection $\sigma : I \to J$, a ***linear*** *map $\boldsymbol{P}[\sigma] : \boldsymbol{P}[I] \to \boldsymbol{P}[J]$.*

Definition 4 *A* ***connected Hopf monoid in vector species*** $\boldsymbol{H}$ *consists of:*

a) a vector species $\boldsymbol{H}$ s.t. $\boldsymbol{H}[\emptyset] = \Bbbk$ (connectednes)
b) linear *maps*

$$\boldsymbol{H}[S] \otimes \boldsymbol{H}[T] \xrightarrow{\mu_{S,T}} \boldsymbol{H}[I] \quad \textit{and} \quad \boldsymbol{H}[I] \xrightarrow{\Delta_{S,T}} \boldsymbol{H}[S] \otimes \boldsymbol{H}[T]$$

for each decomposition $I = S \sqcup T$.

*Moreover, the linear maps should satisfy the **axioms** above.*

Definition 5 *Let* $\boldsymbol{H}$ *be a connected Hopf monoid in vector species. The **antipode** of* $\boldsymbol{H}$ *is the collection of maps*

$$s_I : \boldsymbol{H}[I] \to \boldsymbol{H}[I],$$

one for each finite set I*, given by* $s_\emptyset = \mathrm{id}$ *and*

$$s_I = \sum_{\substack{I=S_1\sqcup\cdots\sqcup S_k,\\ k\geq 1,\ S_i\neq\emptyset}} (-1)^k \mu_{S_1,\ldots,S_k} \circ \Delta_{S_1,\ldots,S_k} \quad \textit{for } I \neq \emptyset$$

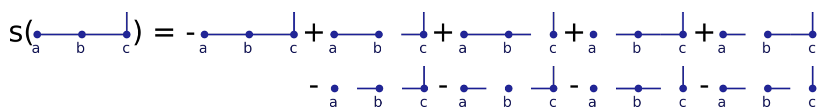


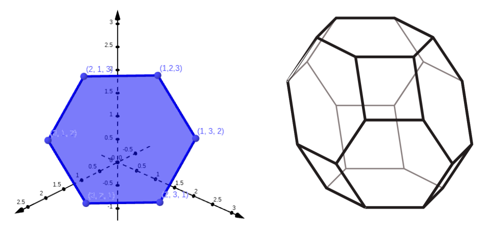
Figure 14: Example of an antipode formula for graphs

Generalized permutahedra

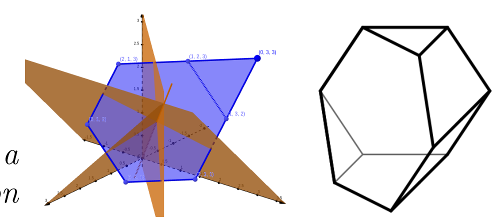
Let $\mathbb{R}I$ be the real vector space with basis I. Use e_i for basis vector in $\mathbb{R}I$ and i for element in I.

Definition 6 *The **standard permutahedron*** π_n *is the convex hull of the* $n!$ *permutations of* $I = [n] = \{1, \ldots, n\}$*, i.e.*

$$\pi_I := \mathrm{conv}\{(\omega(1), \ldots, \omega(n) : \omega \in S_n\} \subset \mathbb{R}I$$



Definition 7 (Generalized permutahedron)
informally: *deformation of standard permutahedron*
   a) move vertices while preserving edge directions
   b) translate facets without passing vertices
formally: *normal fan of generalized permutahedra is a* coarsening *of the normal fan of the standard permutahedron* π_I



Some subclasses of generalized permutahedra: graphic zonotopes, hypergraphic polytopes, simplicial complex polytopes, nestohedra, associahedra, standard permutahedra, ...

Hopf monoid of generalized permutahdra

For a decomposition $S \sqcup T = I$ and $\mathfrak{p} \in \mathrm{GP}[S]$, $\mathfrak{q} \in \mathrm{GP}[T]$ we have

$$\mathfrak{p} \times \mathfrak{q} \in \mathrm{GP}[I]$$

Define the **product** to be

$$\mathfrak{p} \cdot \mathfrak{q} := \mathfrak{p} \times \mathfrak{q}$$

There exist generalized permutahedra $\mathfrak{p}|_S \subset \mathbb{R}S$ and $\mathfrak{p}/_S \subset \mathbb{R}T$ s.t.

$$\mathfrak{p}_{S,T} = \mathfrak{p}|_S \times \mathfrak{p}/_S$$

Define the **coproduct** to be

$$\Delta_{S,T}(\mathfrak{p}) = (\mathfrak{p}|_S, \mathfrak{p}/_S)$$

Theorem 8 (Antipode) *The antipode of the Hopf monoid* GP *of generalized permutahedra is given by the following* cancellation-free *and* grouping-free *formula.*

For generalized permutahedra $\mathfrak{p} \subset \mathbb{R}I$ *we have*

$$s_I(\mathfrak{p}) = (-1)^{|I|} \sum_{\mathfrak{q} \leq \mathfrak{p}} (-1)^{\dim \mathfrak{q}} \mathfrak{q}$$

where we sum over all nonempty faces $\mathfrak{q}$ *of* $\mathfrak{p}$*.*

Application: Reciprocity results

Theorem 9 (Reciprocity) *Let* $\boldsymbol{H}$ *be a connected Hopf monoid,* $\zeta : \boldsymbol{H}[I] \to \Bbbk$ *be a character and* χ *be the associated polynomial invariant. Let* s *be the antipode of* $\boldsymbol{H}$*. Then*

$$\chi_I(x)(-1) = \zeta_I(s_I(x)),$$

more generally

$$\chi_I(x)(-n) = \chi_I(s_I(x))(n).$$

Theorem 10 (Reciprocity for basic character and generalized permutahedra) *At a natural number* n*, the* basic invariant *of a generalized permutahedron* $\mathfrak{p} \subset \mathbb{R}I$ *is given by*

$$\chi_I(p)(n) = (\#\ \textit{of}\ \mathfrak{p}\textit{-generic functions}\ y : I \to [n])$$

where y *is* $\mathfrak{p}$*-generic iff* $\mathfrak{p}_y$ *is a point. Moreover,*

$$(-1)^I \chi_I(p)(-n) = \sum_{y:I\to[n]} (\#\ \textit{of vertices of}\ \mathfrak{p}_y)$$

where $\mathfrak{p}_y$ *is the* y*-maximum face of* $\mathfrak{p}$*.*

Application to graphs: The basic polynomial invariant is the *chromatic polynomial* of graphs, which equals

$$\chi_I(g)(n) = (\#\text{ of } \textbf{proper} \text{ colorings } y : I \to [n] \text{ of g with } n \text{ colors})$$

and

$$(-1)^I \chi_I(g)(-n) = (\#\text{ of compatible pairs of acyclic orientations of } g \text{ and } n\text{-colorings } y : I \to [n])$$

$$(-1)^I \chi_I(g)(-1) = (\#\text{ of acyclic orientations of } g)$$

(Stanley's reciprocity theorem for graphs, 1973)

Similar results using basic characters for

- Matroids (Billera-Jia-Reiner polynomial, 2009)
- Matroids (Bergmann polynomial reciprocity)
- Posets (Stanley's reciprocity theorem for posets, 1970)
- Hypergraphs (Aval, Karaboghossian, Tanasa, 2018)
- ...

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Presented paper by Kenta Ozeki, Carol T. Zamfirescu

Every 4-connected graph with crossing number 2 is Hamiltonian

(http://http://hdl.handle.net/1854/LU-8585100)

Introduction

A cycle or a path (in graph G)is *Hamiltonian* if it contains all vertices of G. A graph is called *Hamiltonian* if it has a Hamiltonian cycle and *Hamiltonian-connected* if there is a Hamiltonian path between any two vertices in the graph. A cut is set of vertices whose removal disconnects the graph.

In 1931 Whitney[6] proved that 4-connected planar triangulations are Hamiltonian and later Tutte[5] extended this to all 4-connected planar graphs. Ozeki and Zamfirescu have strengthend results on the Hamiltonicity of 3-connected planar graphs with a specified number of 3-cuts. Brinkmann and Zamfirescu[1] states that every 3-connected planar graph with at most three 3-cuts is Hamiltonian. Jackson and Yu[3] showed that plane triangulations with at most three separating triangles is Hamiltonian.

The two following notions help to classify nonplanar graphs. *Crossing number* $cr(G)$ of graph G is the minimum number of edge crossings over all plane drawings of G. *The genus* of a graph G is the smallest k such that the graph can be embedded on a sphere with k handles. Grünbaum[2] conjectured that every 4-connected graph of genus 1 is Hamiltonian.

Kawarabayashi and Ozeki[4] showed that every 4-connected projective-planar graph is Hamiltonian-connected. Since any graph with crossing number 1 can be embedded into the projective plane, it follows that every 4-connected graph with crossing number at most 1 is Hamiltonian-connected. Brinkmann showed that if e and f are the crossing edges in a 4-connected graph G with crossing number 1, then G contains a Hamiltonian cycle avoiding e and f.

In this paper, authors show the following result:

Theorem 1 *Every* 4*-connected graph with crossing number at most* 2 *is Hamiltonian.*

In this talk, we will present a proof of Theorem 1. First, we will focus on some necessary related results. Afterwards we will discuss some application of Theorem 1. And at the end we give a tabular overview of certain Hamiltonian properties of 3-connected graphs with few crossings and a small number of 3-cuts.

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Presented paper by Yusuke Kobayashi

NP-hardness and fixed-parameter tractability of the minimum spanner problem

(https://www.sciencedirect.com/science/article/pii/S030439751830447X)

Introduction

Definition 1 *For a positive integer t, a* t-spanner *of a graph G is a spanning subgraph H such that the distance between every pair of vertices in H is at most t times of that in G.*

That is, a subgraph $H = (V, E_H)$ *of* $G = (V, E)$ *is said to be a t-spanner if* $d_H(u, v) \leq t \cdot d_G(u, v)$ *for any* $u, v \in V$.

We consider the following problem.

Definition 2 *Let t be a positive integer. We call* Minimum t-Spanner Problem *the problem of finding a t-spanner with minimum number of edges in a given graph.*

Minimum t-Spanner Problem is known to be NP-hard in general graphs, and also in planar graphs for $t \geq 5$. We show that it is NP-hard in planar graphs for t in $\{2, 3, 4\}$ (for $t = 1$ it is trivial, since the optimal solution is obtained by removing parallel edges).

We also present a fixed-parameter algorithm for this problem in which the number of removed edges is regarded as a parameter.

NP-hardness

Definition 3 *For a subgraph H of G, the set of edges in H is denoted by* $E(H)$. *For a vertex set* $X \subseteq V$, *let* $\delta_G(X)$*denote the set of all edges in G connecting X and* $V \setminus X$.

Definition 4 *Let t be a positive integer. We call* Dominating Set with Degree-k-Constraint *the problem of finding a minimal dominating set containing every vertex of degree at least k.*

Definition 5 *A graph* $G = (V, E)$ *is nearly k-edge-connected if the minimum degree of G is at least* $k - 1$. *And* $|\delta_G(X)| \leq k - 1 \implies |X| \leq 1$ *or* $|V \setminus X| \leq 1$ *for any* $X \subseteq V$.

Lemma 6 *Let t be a positive integer. For a graph* $G = (V, E)$, *its subgraph* $H = (V, E_H)$ *is a t-spanner if and only if* $d_H(u, v) \leq t$ *for any* $uv \in E \setminus E_H$.

Lemma 7 Dominating Set with Degree-k-Constraint *in nearly k-edge-connected planar graphs is* NP-*hard for* $k \in \{4, 5, 6\}$.

Lemma 8 Dominating Set with Degree-$(t + 2)$-Constraint *in nearly* $(t + 2)$*-edge-connected planar graphs can be reduced to* Minimum t-Spanner Problem *in planar graphs.*

FPT algorithm

Definition 9 *An algorithm parameterized by k is called a fixed-parameter algorithm(or an FPT algorithm) if its running time is bounded by* $f(k) \cdot (|V| + |E|)^{O(1)}$ *for some function f.*

Lemma 10 *For a positive integer t, there exists a FPT algorithm for* Minimum t-Spanner Problem *parameterized by k that runs in* $O\left(k \left(k^2 t \left(t + 1\right)\right)^{k+1} + |V||E|\right)$ *time where k is the number of removed edges.*

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Presented paper by Ian Morrison

Sacks of Dice with Fair Totals

(https://arxiv.org/abs/1411.2272)

Introduction

Having a fair die is cool. You can throw and every outcome will have the same probability. But if you throw with more fair dice not every sum has the same probability. In this talk we examine question: How looks a sack of independent dice (not neccessary fair) with any number of sides, for which all totals are equally likely.

Problem 1 *You can change number of dots on any side of two six-sided dice. Change them in such a way that every result of throw with changed dice has the same probability as throw with standard dice.*

Problem 2 *Find a sack with three dice such that if all dice are thrown every outcome form the set* $\{0, 1, 2, \ldots, 11\}$ *are equally likely.*

Dice can have any number of sides with any probability.

Problem 3 *For every* $k \leq n$ *construct dice that have all sums in* $\{0, 2, \ldots 2^n - 1\}$ *equaly likely.*

Definitions

Definition 4 *A die* d *of order* $n \geq 2$ *is a finite probability space whose sample space is the set* $\langle n \rangle = \{0, 1, 2, \ldots, n-1\}$.

Probability of side with number j *is* $p_d(j)$.

Definition 5 *Dice* d *is* ***semifair*** *if*

- *each* $p_d(j)$ *is either* 0 *or equal to* $p_d(0)$*, which is nonzero,*
- *it is* ***palindromic****:* $p_d(n-j-1) = p_d(j)$.

Definition 6 *A partition* Π *is interval free if no part contains consecutive elements of* $[l]$.

Theorems

Theorem 7 *A sack is fair if and only if*

- *each die in it is semifair,*
- *each total is obtained from a unique effective roll.*

Theorem 8 *Every partition-factorization sack* S *arises from an interval free partition of an ordered factorization, both of which are uniquely determined by* S.

Theorem 9 *Every fair sack* S *of size* m *and total* t *equals* $S_{a,\Pi}$ *for* Π *a uniquely determined iterval free partition with* m *parts of an ordered factorization* a *of* t.

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Presented paper by Rennan Dantasa, Rudini M. Sampaioa, Frederic Havet

Minimum density of identifying codes of king grids

(http://link.springer.com/article/10.1007%2FBF02579200)

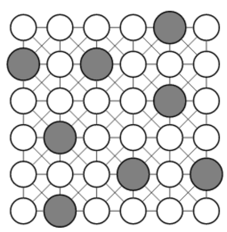
Introduction

The problem of finding low-density identifying codes was introduced in relation to fault diagnosis in arrays of processors. The aim is to find a small subset of vertices C such that every vertex is uniquely identified by his neighbours from C. Most interest was dedicated to graphs with a grid topology. This paper studies the problem for *king grids*.

Definition 1 *Graph G is a* king grid *if it is a strong product of two paths P_1 and P_2 ($G = P_1 \boxtimes P_2$).*

Definition 2 *Graph K_k is a* king strip *if $K_k = P_{\mathbb{N}} \boxtimes P_k$.*

Definition 3 *Let $G = (V, E)$ be a graph and $N[v]$ denote closed neighbourhood of $v \in V$. For subset $C \subseteq V$, let $C[v] = N[v] \cap C$. Subset C is an* identifying code *if $C[v] \neq \{\}$ and $C[v] \neq C[u]$ for all distinct $u, v \in V$.*



Because G can be infinite we define *density* of C in G a little bit more carefully as

$$d(C, G) = \limsup_{r \to \infty} \frac{|C \cap B_r(v_0)|}{|B_r(v_0)|}$$

where $B_r(v_0)$ is a ball of radius r around an arbitrary vertex v_0. The infimum of the density of an identifying code in G is denoted by $d^\star(G)$.

Results

Using the discharging method we prove the following theorems:

Theorem 4 *If G is a (finite or infinite) king grid, then $d^\star(G) \geq \frac{2}{9}$.*

Theorem 5 *If G is a finite king grid, then $d^\star(G) > \frac{2}{9}$.*

Theorem 6 *If K_k is a king grid and $k \geq 6$, then $d^\star(K_k) > \frac{2}{9} + \frac{8}{81k}$.*

Theorem 7 *For $k \geq 5$:*

$$d^\star(K_k) \leq \begin{cases} \frac{2}{9} + \frac{6}{18k} & \text{if } k \equiv 0 \mod 3 \\ \frac{2}{9} + \frac{8}{18k} & \text{if } k \equiv 1 \mod 3 \\ \frac{2}{9} + \frac{7}{18k} & \text{if } k \equiv 2 \mod 3 \end{cases}$$

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