3. DÚ

(a_n) nemá limitu

- chceme pokrýt nekonečně podposloupnostmi s limitou $a \in \mathbb{R}^*$

(a'_n) podposloupnost (a_n) s limitou $a \in \mathbb{R}^*$

$\forall i \in (a_n)$ i není v (a'_n) vytvoříme $(a^{(i)}_n) = (a'_n) \cup \{i\}$

1. $\sum_{n=1}^{\infty} \frac{n}{2^n n^2 + 15}$ $\quad s_k = \sum_{n=1}^{k} \frac{n}{2^n n^2+15} \le \sum_{n=1}^{k} \frac{1}{2^n n + \frac{15}{n}} \le \sum_{n=1}^{k} \frac{1}{2^n} = 1$

2. $\sum_{n=1}^{\infty} \frac{1}{n(n+k)}$ $\quad k \in \mathbb{Z}^+_0$ $\quad \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+k)} = \frac{1}{k} \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+k} \right) =$$

$$= \frac{1}{k} \sum_{n=1}^{k} \frac{1}{n} = \frac{1}{k} H_k \qquad H_k := \sum_{n=1}^{k} \frac{1}{n}$$

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f JE DEFINOVANÁ NA OKOLÍ BODU b $(U(b,\delta))$

DERIVACE f V BODĚ b JE $f'(b) := \lim_{h \to 0} \frac{f(b+h) - f(b)}{h} = \lim_{x \to b} \frac{f(x)-f(b)}{x-b}$

$$\left(\frac{1}{x^2}\right)' = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim \frac{x^2 - (x+h)^2}{(x+h)^2 x^2} \cdot \frac{1}{h} =$$

$$= \lim \frac{-2xh - h^2}{(x+h)^2 x^2} \cdot \frac{1}{h} = \lim \frac{-2x - h}{x^2 (x+h)^2} = -\frac{2x}{x^4} = -\frac{2}{x^3}$$

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1. $(x^n)' = n x^{n-1}$ $\quad$ 3. $(\ln(x))' = \frac{1}{x}$ $\quad$ 5. $(\cos(x))' = -\sin(x)$

2. $(e^x)' = e^x$ $\quad$ 4. $(\sin(x))' = \cos(x)$

$(\alpha f + \beta g)' = \alpha f' + \beta g'$ $\qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$(fg)' = f'g + fg'$ $\qquad (f(g))' = f'(g) g'$

$$(x^2 e^{-x^2})' = 2x e^{-x^2} + x^2 \left(e^{-x^2} \cdot (-2x)\right) = (1 - x^2)\, 2x e^{-x^2}$$

$\frac{\ln(1+x)}{x}$ $\qquad (f)'$? $\qquad f = \begin{cases} \frac{\ln(1+x)}{x} & \text{PRO } x \in (-1,\infty) \setminus \{0\} \\ 1 & \text{PRO } x = 0 \end{cases}$

$$\left(\frac{\ln(1+x)}{x}\right)' = \frac{\frac{1}{1+x} \cdot x - \ln(1+x)}{x^2} = \frac{\frac{x}{1+x} - \ln(1+x)}{x^2}$$

$$f'(0) = \lim_{h \to 0} \frac{\frac{\ln(1+0+h)}{h} - f(0)}{h} = \lim_{h \to 0} \frac{\frac{\ln(1+h)}{h} - 1}{h} =$$

$$= \lim_{h \to 0} \frac{\ln(1+h) - h}{h^2} = \lim_{h \to 0} \frac{\frac{1}{1+h} - 1}{2h} = \lim \frac{-\frac{1}{(1+h)^2}}{2} = -\frac{1}{2}$$

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l'HOSPITALOVO PRAVIDLO

$f, g : P(a,\delta) \to \mathbb{R}$ MAJÍ NA $P(a,\delta)$ VLASTNÍ DERIVACE A $g'(x) \neq 0$ NA $P(a,\delta)$

1. $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ A $\lim_{x \to a} \frac{f'(x)}{g'(x)} = A \in \mathbb{R}^*$, POTOM

$\lim_{x \to a} \frac{f(x)}{g(x)} = A$

2. $\lim_{x \to a} |g(x)| = +\infty$ A $\lim_{x \to a} \frac{f'(x)}{g'(x)} = A \in \mathbb{R}^*$, PAK $\lim_{x \to a} \frac{f(x)}{g(x)} = A$

$$\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right) = \lim_{x \to \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x \to \infty} \frac{-\frac{\cos\left(\frac{1}{x}\right)}{x^2}}{-\frac{1}{x^2}} = \lim_{x \to \infty} \cos\left(\frac{1}{x}\right) = 1$$

$$\lim_{x \to \infty} \frac{\ln(x^8 + x^3 + 8x + 4)}{\ln(x^5 + 2x^4)} = \lim_{x \to \infty} \frac{\frac{8x^7 + 3x^2 + 8}{x^8 + x^3 + 8x + 4}}{\frac{5x^4 + 8x^3}{x^5 + 2x^4}} = \lim_{x \to \infty} \frac{(8x^7 + 3x^2 + 8)(x^5 + 2x^4)}{(x^8 + x^3 + 8x + 4)(5x^4 + 8x)} = \frac{8}{5}$$

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n, n^2 $\qquad \lim_{n \to \infty} \frac{n}{n^2}$ $\qquad \lim_{n \to \infty} \frac{\ln(n)}{n} = 0$

$x^{(x^x)}$ $\qquad x^x$ např. pro $x = -\frac{1}{2}$ $\quad \left(-\frac{1}{2}\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{-\frac{1}{2}}}$ nedává smysl

většinou uvažujeme pouze $\mathbb{R}^+$ jako definiční obor

$$(x^x)' = \left(e^{x \ln x}\right)' = e^{x \ln x} (\ln x + 1) = x^x (\ln x + 1)$$

$$\left(x^{(x^x)}\right)' = \left(e^{x^x \ln x}\right)' = x^{(x^x)} \left(x^x (\ln x + 1) \ln x + x^{x-1}\right) = x^{(x^x) + x - 1} \left(x \ln^2 x + x \ln x + 1\right)$$

$\sqrt{\frac{x-1}{x+1}}$ $\qquad \frac{x-1}{x+1} \ge 0$

	-1		1	
$x-1$	$-$	$-$		$+$
$x+1$	$-$	$+$		$+$

DO je $(-\infty, -1) \cup [1, \infty)$

$$\frac{1}{\cancel{2}\sqrt{\frac{x-1}{x+1}}} \cdot \frac{\cancel{x} + \cancel{1} - (\cancel{x} - \cancel{1})}{(x+1)^2} = \frac{1}{(x+1)^2 \sqrt{\frac{x-1}{x+1}}} = \frac{1}{\sqrt{(x-1)(x+1)^3}} = \frac{1}{(x+1)\sqrt{x^2-1}}$$

$\arcsin(x)$ je inv. pro sin na $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\to$ DO $\sin\left(\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right) = [-1, 1]$

Derivace inverzní funkce

$J \subseteq \mathbb{R}$ je interval $\quad a \in J$ vnitřní bod $\quad f : J \to \mathbb{R}$ je spojitá a ryze mon.

$f(a) = b$. Potom $(f^{-1})'(b) = \frac{1}{f'(a)}$ $\qquad a = \arcsin(x)$, $b = x$

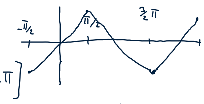
pro $x \in (-1, 1)$ platí $\sin(\arcsin(x)) = x$ $\qquad \sin^2(x) + \cos^2(x) = 1$

$\cos(x) = \sqrt{1 - \sin^2(x)}$

$$\arcsin(x)' = \frac{1}{\sin(\arcsin(x))'} = \frac{1}{\cos(\arcsin(x))} =$$

$$= \frac{1}{\sqrt{1 - \sin^2(\arcsin(x))}} = \frac{1}{\sqrt{1 - x^2}}$$

$\arcsin(\sin(x))$ $\qquad$ DO $\mathbb{R}$



$x \in \left[-\frac{\pi}{2} + 2k\pi, +\frac{\pi}{2} + 2k\pi\right]$

se chová jako $x - 2k\pi$ $\Rightarrow$ derivace je 1 $\left(-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right)$

$x \in \left[\frac{\pi}{2} + 2k\pi, \frac{3}{2}\pi + 2k\pi\right]$

se chová jako $-x - 2k\pi$ $\Rightarrow$ derivace je -1 $\left(\frac{\pi}{2} + 2k\pi, \frac{3}{2}\pi + 2k\pi\right)$