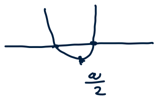
1. DÚ

1. $\{a \in \mathbb{R};\ (\forall x \in \mathbb{R})\ (|x-2| \le 1 \Rightarrow x^2 - ax > 5)\} = (-\infty, -4)$

$x \in [1, 3]$



PRO $x = 1$

$1 - a > 5$

$a < -4$

2. $\lim\limits_{n\to\infty} \dfrac{(4n-10)^{80}(3n+15)^{10}}{(4n-16)^{90}} = \lim\limits_{n\to\infty} \dfrac{n^{90}\left(4-\frac{10}{n}\right)^{80}\left(3+\frac{15}{n}\right)^{10}}{n^{90}\left(4-\frac{16}{n}\right)^{90}}$

$= \dfrac{4^{80}\cdot 3^{10}}{4^{90}} = \left(\dfrac{3}{4}\right)^{10}$

$\lim\limits_{n\to\infty} \dfrac{3n^2+\sqrt{n}}{(4n-16)^5} \to 0$

$\lim\limits_{n\to\infty} (-1)^n 10 + n$

ZDOLA $-10+n$ $\searrow \infty$

SHORA $10+n$ $\nearrow$

2 POLICAJTI

$\mathcal{U}(x,\delta) := (x-\delta, x+\delta)$

$P(x,\delta) := \mathcal{U}(x,\delta) \setminus \{x\}$

$\lim\limits_{x\to a} f(x) = A \in \mathbb{R}^*$ POKUD $\forall \varepsilon > 0\ \exists \delta > 0: x \in P(a,\delta) \Rightarrow f(x) \in \mathcal{U}(A,\varepsilon)$

$\lim\limits_{x\to 0} \sin\left(\frac{1}{x}\right)$ NEMÁ LIMITU ANI ZPRAVA, ANI ZLEVA

HEINEHO DEFINICE LIMITY

1. $\lim\limits_{x\to a} f(x) = A$
2. $\forall$ POSL. $(x_n) \subseteq P(a,\Delta);\ \lim\limits_{n\to\infty} x_n = a$ PLATÍ, ŽE $\lim\limits_{n\to\infty} f(x_n) = A$

ZNÁMÉ LIMITY

1. $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x} = 1$ 2. $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x} = 1$

$\lim\limits_{x\to\infty} \dfrac{1}{x-12} \to 0$ $\varepsilon > 0$, CHCEME $\delta > 0$; $\forall x > \delta: \left|\dfrac{1}{x-12}\right| < \varepsilon$

$\dfrac{1}{x-12} < \varepsilon \iff \dfrac{1}{\varepsilon} + 12 < x$

$\delta := \dfrac{1}{\varepsilon} + 12$

$\lim\limits_{x\to 1} \dfrac{x-1}{x-1} = 1$

$\lim\limits_{x\to 1} \dfrac{x-1}{x-1} = \lim\limits_{x\to 1} 1$ $\dfrac{x-1}{x-1} \ne 1$

$\lim\limits_{x\to 1} \dfrac{x^2+2x-3}{x^2-1} = \lim\limits_{x\to 1} \dfrac{(x+3)(x-1)}{(x+1)(x-1)} = \lim\limits_{x\to 1} \dfrac{x+3}{x+1} \lim\limits_{x\to 1} \dfrac{x-1}{x-1} =$

$= \lim\limits_{x\to 1} \dfrac{x+3}{x+1} \cdot 1 = 2\cdot 1 = 2$

$\lim\limits_{x\to\infty} \dfrac{\ln(1+2^x)}{x} = \ln(2)$

$\underset{\downarrow\ \ln(2)}{\ln(2)} \text{ ,,=''} \dfrac{\ln(2^x)}{x} \le \dfrac{\ln(1+2^x)}{x} \le \dfrac{\ln(2^x+2^x)}{x} = \underbrace{\dfrac{\ln(2)+x\ln(2)}{x}}_{\downarrow\ \ln(2)}$

2 POLICAJTI + HEINE

// $\ln(2^x+2^x) = \ln(2\cdot 2^x) = \ln(2) + x\ln(2)$

$\lim\limits_{x\to 0} \dfrac{1-\cos(x)}{x^2} =$

$= \lim\limits_{x\to 0} \dfrac{1-\cos(x)}{x^2}\,\dfrac{1+\cos(x)}{1+\cos(x)} = \lim\limits_{x\to 0} \dfrac{1-\cos^2(x)}{x^2(1+\cos(x))} = \lim \dfrac{\sin^2(x)}{x^2(1+\cos(x))} =$

$= \lim\limits_{x\to 0} \underset{\searrow 1}{\dfrac{\sin(x)}{x}} \lim\limits_{x\to 0} \underset{\searrow 1}{\dfrac{\sin(x)}{x}} \lim\limits_{x\to 0} \underset{\searrow \frac{1}{2}}{\dfrac{1}{1+\cos(x)}} = \dfrac{1}{2}$

VOLSF

$A, B, C \in \mathbb{R}^*$, $g(x)$ je fce; $\lim\limits_{x\to A} g(x) = B$,

$f(x)$ je fce; $\lim\limits_{x\to B} f(x) = C$,

PAK $\lim\limits_{x\to A} f(g(x)) = C$, POKUD JE SPLNĚNA P1 NEBO P2

P1 fce $f(x)$ je spojitá v B $(f(B) = C)$

P2 na nějakém prstencovém okolí $P(A,\eta)$ $g(x)$ nenabývá B, tzn. $B \notin g(P(A,\eta))$

ZNÁME $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x} = 1$

$\lim\limits_{x\to 0} (1+4x)^{\frac{1}{3x}} = \lim\limits_{x\to 0} e^{\frac{1}{3x}\ln(1+4x)} = \lim\limits_{x\to 0} e^{\underbrace{\frac{4x}{3x}\underbrace{\frac{\ln(1+4x)}{4x}}_{\to 1 \text{ VOLSF}}}}$