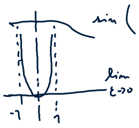
$\mathrm{sgn}(x)$ nemá Newtonův integrál $(N)\int_{-1}^{1} \mathrm{sgn}(x)\,dx$, ale existuje $(R)\int_{-1}^{1}\mathrm{sgn}(x)\,dx = 0$

$f(x) = \dfrac{x^2}{\sqrt{1-x^2}}$ na $(-1,1)$ nemá Riemannův

$$(N)\int_{-1}^{1} \frac{x^2}{\sqrt{1-x^2}}\,dx = (N)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2(t)}{\sqrt{\underbrace{1-\sin^2(t)}_{\cos^2(t)}}}\cdot \cancel{\cos(t)}\,dt =$$

$x = \sin(t)$

$$= (N)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2(t)\,dt = \frac{1}{2}\Big[t - \cos(t)\sin(t)\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi}{2}$$



$\sin\big((0,\pi)\big) = (0,1]$

$$\lim_{\varepsilon\to 0}\int_0^{1-\varepsilon} f(x)\,dx + \lim_{\varepsilon\to 0}\int_{-1+\varepsilon}^{0} f(x)\,dx$$

Nevlastní integrál

$$\int_0^\infty \frac{dx}{1+x^2} = \lim_{a\to\infty}\int_0^a \frac{dx}{1+x^2} = \lim_{a\to\infty}\big[\mathrm{arctg}(x)\big]_0^a =$$

$$= \lim_{a\to\infty} \mathrm{arctg}(a) - \underbrace{\mathrm{arctg}(0)}_{=0} = \frac{\pi}{2}$$

$$\int_0^\infty \cos(x)\,dx = \lim_{a\to\infty}\int_0^a \cos(x)\,dx = \lim_{a\to\infty}\big[\sin(x)\big]_0^a =$$

$= \lim_{a\to\infty} \sin(a) - 0$ nemá limitu — Heine + umíme najít podposloupnosti s limitami -1 a 1

$$\int_1^\infty \frac{\ln(x)}{x}\,dx = \lim_{a\to\infty}\int_{\ln(1)=0}^{\ln(a)} t\,dt = \lim_{a\to\infty}\frac{1}{2}\big[t^2\big]_0^{\ln(a)} = \lim_{a\to\infty}\ln^2(a) = \infty$$

$\displaystyle\int_{-\infty}^{\infty}\frac{dx}{4+x^2} = \frac{\pi}{4}$ rozdělíme na 2 integrály

$$\lim_{a\to\infty}\int_0^a \frac{dx}{4+x^2} = \lim_{a\to\infty}\frac{1}{4}\int_0^a \frac{dx}{1+\left(\frac{x}{2}\right)^2} = \lim_{a\to\infty}\frac{1}{2}\left[\mathrm{arctg}\left(\frac{x}{2}\right)\right]_0^a = \frac{\pi}{4}$$

stejně $\displaystyle\lim_{a\to\infty}\int_{-a}^{0}\frac{dx}{4+x^2} = \frac{\pi}{2}$

gama funkce

$$\Gamma(n) := \int_0^\infty t^{n-1}e^{-t}\,dt$$

Pokud $n\in\mathbb{N}$

$$\int_0^\infty t^{n-1}e^{-t}\,dt = \underbrace{\Big[-t^{n-1}e^{-t}\Big]_0^\infty}_{=0} + (n-1)\int_0^\infty t^{n-2}e^{-t}\,dt = (n-1)(n-2)\int_0^\infty t^{n-3}e^{-t}\,dt =$$

$$= \dots = (n-1)!\int_0^\infty e^{-t}\,dt = (n-1)!$$

$f(x) = \sqrt{r^2-x^2}$

objem $\displaystyle\pi\int_{-r}^{r}\left(\sqrt{r^2-x^2}\right)^2dx = \pi\int_{-r}^{r} r^2-x^2\,dx = \pi\left[xr^2 - \frac{1}{3}x^3\right]_{-r}^{r} = \frac{4}{3}\pi r^3$

$$f'(x) = -\frac{2x}{2\sqrt{r^2-x^2}} = -\frac{x}{\sqrt{r^2-x^2}}$$

povrch $\displaystyle 2\pi\int_{-r}^{r}\sqrt{r^2-x^2}\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx \;\text{„=“}\; 2\pi\int_{-r}^{r} r\,dx = 2\pi\big[rx\big]_{-r}^{r} = 4\pi r^2$

VZOROVÝ ZÁPOČTOVÝ TEST - ÚLOHA 1

$\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{n^2+k}}$ 2 POLICAJTI

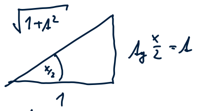
$$1 \leftarrow \frac{n}{n} = \sum_{k=1}^{n}\frac{1}{\sqrt{n^2}} \geq \sum_{k=1}^{n}\frac{1}{\sqrt{n^2+k}} \geq \sum_{k=1}^{n}\frac{1}{\sqrt{n^2+n}} = \frac{n}{\sqrt{n^2+n}} \to 1$$

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{n^2+k}} = 1$$

GONIOMETRICKÉ SUBSTITUCE

$$\int R(\sin x, \cos x)\,dx$$

$\mathrm{tg}\,\frac{x}{2} = t \quad x\in(-\pi,\pi)$



$\sin\frac{x}{2} = \dfrac{t}{\sqrt{1+t^2}}$, $\cos\frac{x}{2} = \dfrac{1}{\sqrt{1+t^2}}$ $\qquad \sin 2\varphi = 2\sin\varphi\cos\varphi$, $\cos 2\varphi = \cos^2\varphi - \sin^2\varphi$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} = \frac{2t}{1+t^2}$$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \left(\frac{1}{\sqrt{1+t^2}}\right)^2 - \left(\frac{t}{\sqrt{1+t^2}}\right)^2 = \frac{1-t^2}{1+t^2}$$

sin x

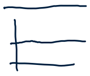
$$\int \frac{\cancel{1+t^2}}{\cancel{2}t} \cdot \frac{\cancel{2}}{\cancel{1+t^2}}\,dt = \int \frac{1}{t}\,dt = \ln|t| + C = \ln\left|\operatorname{tg}\frac{x}{2}\right| + C$$

$$\int \frac{1}{\cos x - 2\sin x + 3}\,dx \qquad \sin x = \frac{2t}{1+t^2}$$

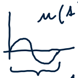
$$\int \frac{1}{\frac{1-t^2}{1+t^2} - 2\frac{2t}{1+t^2} + 3} \cdot \frac{2}{1+t^2}\,dt = \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad dx = \frac{2}{1+t^2}\,dt$$

$$= \int \frac{2}{\frac{-t^2-4t+1+3+3t^2}{\cancel{1+t^2}}} \cdot \frac{1}{\cancel{1+t^2}}\,dt = \int \frac{1}{\underbrace{t^2-2t+1}_{(t-1)^2}+1}\,dt =$$

$$= \int \frac{1}{1+(t-1)^2}\,dt = \operatorname{arctg}(t-1) + C = \operatorname{arctg}\left(\operatorname{tg}\frac{x}{2} - 1\right) + C$$



$$\frac{U^2}{R}$$



$$u(t) = A \cdot \sin(2\pi \cdot 50\, t)$$

$$T = \frac{1}{50}$$

$$\frac{U_{ef}^2}{\cancel{R}} = \frac{\int_0^T \frac{u^2(t)}{\cancel{R}}\,dt}{T} =$$

$\sin^2$

$$x = 2\pi\, 50\, t$$
$$dx = 2\pi\, 50\, dt$$

$$= \int_0^{2\pi} \frac{A^2 \cdot \sin^2(x)}{2\pi}\,dx =$$

$$= \int_0^{\pi} \frac{A^2 \cdot \sin^2(x)}{\pi}\,dx = \frac{A^2}{\pi}\int_0^{\pi} \sin^2(x)\,dx = \frac{A^2}{\pi}\left[\frac{1}{2}\left(x + \cos(x)\sin(x)\right)\right]_0^{\pi} =$$

$$= \frac{A^2}{2}$$

$$U_{ef} = \frac{A}{\sqrt{2}}$$ Platí pro harmonický průběh

– vztah je dán jinak