$0 \cdot \infty = \cancel{0}$ NENÍ DEFINOVANÉ !

$\underset{\infty}{x^2} \cdot \underset{0}{\frac{1}{e^x}} = 0$ $\quad$ $\underset{\infty}{x^2} \cdot \underset{0}{\frac{1}{\sqrt{x}}} = \infty$

$\int |x^2 - 1| \, dx$

- $(-\infty, -1)$ $\quad \int x^2 - 1 \, dx = \frac{x^3}{3} - x + c_1$
- $(-1, 1)$ $\quad \int -x^2 + 1 \, dx = -\frac{x^3}{3} + x + c_2$
- $(1, \infty)$ $\quad \int x^2 - 1 \, dx = \frac{x^3}{3} - x + c_3$

Chceme $-\frac{1}{3} + 1 + c_1 = \frac{1}{3} - 1 + c_2$

$-\frac{1}{3} + 1 + c_2 = \frac{1}{3} - 1 + c_3$

$\Updownarrow$

$c_1 - c_2 = -\frac{4}{3}$

$c_2 - c_3 = -\frac{4}{3}$

Na příklad $c_2 = 0 \Rightarrow c_1 = -\frac{4}{3}$

$c_3 = \frac{4}{3}$

$\int \frac{2x+5}{x^3 - 6x^2 - 6x - 7} \, dx$ $\qquad (x - \lambda_1)(x - \lambda_2)(\quad) \dots (x - \lambda_n)$

$$\frac{2x+5}{(x-7)(x^2+x+1)} = \frac{A}{x-7} + \frac{Bx + C}{x^2+x+1} =$$

$$\left[\begin{array}{l} Ax^2 + Ax + A + Bx^2 - 7Bx + Cx - 7C = 2x + 5 \\ A + B = 0 \\ A - 7B + C = 2 \qquad A = \frac{1}{3} \quad B = -\frac{1}{3} \quad C = -\frac{2}{3} \\ A - 7C = 5 \end{array}\right]$$

$$= \frac{1}{3} \frac{1}{x-7} - \frac{1}{3} \frac{x+2}{x^2+x+1}$$

$$\underbrace{\frac{1}{3}\int \frac{1}{x-7} \, dx}_{\frac{1}{3} \ln(x-7)} - \frac{1}{3} \int \frac{x+2}{x^2+x+1} \, dx$$

$$\int \frac{x+2}{x^2+x+1} \, dx = \underbrace{\frac{1}{2} \int \frac{2x+1}{x^2+x+1} \, dx}_{\frac{1}{2} \ln(x^2+x+1)} + \frac{3}{2} \int \frac{1}{x^2+x+1} \, dx$$

$t = x^2 + x + 1$

$dt = 2x + 1$

$$\int \frac{1}{x^2+x+1} \, dx = \int \frac{1}{x^2 + x + \frac{1}{4} + \frac{3}{4}} \, dx =$$

$$= \frac{4}{3} \int \frac{1}{\frac{4}{3}(x + \frac{1}{2})^2 + 1} \, dx = \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \underbrace{\int \frac{2/\sqrt{3}}{\left(\frac{2x+1}{\sqrt{3}}\right)^2 + 1} \, dx}_{\int \frac{1}{1+t^2} \, dt = \operatorname{arctg} t} =$$

$t = \frac{2x+1}{\sqrt{3}}$ $\quad dt = \frac{2}{\sqrt{3}} \, dx$

$$= \frac{2}{\sqrt{3}} \operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)$$

$$(N)\int_0^1 \cos^3(x) \sin(x) \, dx = -(N)\int_{\cos(0)}^{\cos(1)} t^3 \, dt = (N)\int_{\cos(1)}^{\cos(0) = 1} t^3 \, dt = \frac{1}{4} \left[t^4 \right]_{\cos(1)}^{1} = \frac{1}{4} \left(1 - \cos^4(1)\right)$$

$t = \cos x$

$dt = -\sin(x) \, dx$

$$(N)\int_0^1 \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx = \left[\cos\left(\frac{1}{x}\right) \right]_0^1 = \cos(1) - \lim_{x \to 0^+} \cos\left(\frac{1}{x}\right)$$

Problém $\forall \varepsilon > 0 \ \exists a, b \in (0, \varepsilon) ; \cos \frac{1}{a} = 1 \ \& \cos \frac{1}{b} = 0$

$\Rightarrow \cos\left(\frac{1}{x}\right)$ nemá lim zprava v 0

$\Rightarrow (N)\int_0^1 \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$ neexistuje

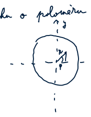
---

Substituční metoda a LA

Hledáme obsah kruhu o poloměru R

$x^2 + y^2 \leq R$

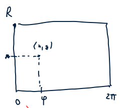
$$\iint_{x^2 + y^2 \leq R} 1 \, dx \, dy$$



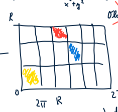
substituce do polárních souřadnic $\longrightarrow$

$r = x^2 + y^2$

$\varphi = \arcsin \frac{y}{x^2 + y^2}$



~~Obsah je~~ $2\pi R$



$\longleftarrow$

$x = r \cos \varphi$

$y = r \sin \varphi$

Musíme uvážit lokální změnu objemu !

$$J = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{pmatrix} = \begin{pmatrix} \cos \varphi & -r \sin \varphi \\ \sin \varphi & r \cos \varphi \end{pmatrix}$$

Su nám popisuje determinant

$$\int_0^{2\pi} \int_0^R 1 \cdot \det(J) \, dr \, d\varphi =$$

$$= \int_0^{2\pi} \int_0^R r \, dr \, d\varphi = \int_0^{2\pi} \frac{R^2}{2} \, d\varphi = \pi R^2$$